Prescribed-Time Adaptive and Optimal Control for a Class of Nonlinear Systems

Thesis submitted in partial fulfillment
for the award of degree

Doctor of Philosophy

by

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2023

In loving memory of my late father and to my beloved mother, who have been my guiding lights throughout my academic journey.

CERTIFICATE

It is certified that the work contained in the thesis titled **Prescribed-Time Adaptive and Optimal Control for a Class of Nonlinear Systems** by **Vijay Kumar Singh** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree. It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. Degree.

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DECLARATION

I, **Vijay Kumar Singh**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. Shyam Kamal** and **Dr. Sandip Ghosh** from July-2019 to Aug.-2023, at the Departement of Electrical Engineering, Indian Institute of Technology (BHU) Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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It is certified that the above statement made by the student is correct to the best of my/our knowledge.

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Acknowledgments

While my name may be the only one on the cover of this dissertation, I am acutely aware that its production would not have been possible without the support and contributions of numerous remarkable individuals. My heartfelt gratitude goes out to all those who have played a significant role in making this thesis a reality. Their guidance, encouragement, and assistance have shaped my post-graduate experience into an unforgettable journey that I will forever cherish.

I seize this moment to convey my profound gratitude and utmost respect to my supervisors, Dr. Shyam Kamal, Associate Professor, and Dr. Sandip Ghosh, Associate Professor, from the Department of Electrical Engineering, IIT (BHU) Varanasi. Their exceptional guidance, continuous monitoring, and unwavering encouragement have been instrumental throughout the entire process of this dissertation. I am truly grateful for their mentorship, which has significantly contributed to the successful completion of this work.

Thank you, Dr. Shyam Kamal, for acknowledging my potential and steering me towards crucial areas of study. Your valuable suggestions, encouraging words, and motivational support during challenging times have been indispensable to me. Your intuition and insights into the problems we faced have resulted in some fascinating discoveries. I am genuinely appreciative of the extensive and impactful discussions we have had, as they have played a pivotal role in shaping my understanding. I am forever thankful for your contributions to my academic journey.

I am immensely grateful to Dr. Sandip Ghosh for his invaluable guidance, unwavering support, and insightful reminders that pursuing a Ph.D. demands patience and dedication, as true achievements cannot be rushed. His encouragement has been a driving force behind my progress, and I sincerely thank him for his mentorship throughout this journey.

I would like to extend my gratitude to Dr. Soumya Ranjan Mohanty from the

Department of Electrical Engineering and Dr. Lavanya Selvaganesh from the Department of Mathematical Sciences, both associated with IIT (BHU) Varanasi. Their valuable comments and suggestions during the evaluation of my research progress are sincerely appreciated.

I express my sincere appreciation to Prof. Bijnan Bandyopadhyay IIT Jodhpur for offering essential insights and invaluable suggestions while preparing certain manuscripts related to my thesis. Gratitude is also extended to Dr. Thach N. Dinh, Associate Professor, CNAM France, for their collaboration, valuable suggestions, and guidance, all of which have substantially enhanced the quality of our research. Your support and contributions have played a crucial role in shaping the outcome of our work, and I genuinely value your input.

I am deeply grateful to the staff members, including Mr. A. N. Singh, Mr. S. K. Maurya, and Mr. Anjneya, of the Electrical Engineering Department at IIT (BHU) Varanasi, for their invaluable support. I extend my heartfelt thanks to my seniors, Dr. A. Sachan, Dr. A. K. Pal, and Dr. B. Singh for their valuable collaborations, technical discussions, and suggestions that have greatly enriched my research. Special appreciation goes to Mr. K. Chatterjee for his invaluable assistance and efforts in fostering collaborations. Lastly, I am truly thankful to Dr. S. K. Soni and Dr. S. Kumar for their unwavering encouragement and support, which have played a vital role in my academic and research journey.

I extend heartfelt and special gratitude to my colleagues and friends, particularly acknowledging Mr. P. Prasun, Mrs. R. Singh, Mrs. H. Mittapally, and Mr. A. Kumar. Throughout this research journey, your unwavering support and encouragement during my challenging moments have been truly invaluable. Your presence has made this experience more meaningful and enjoyable. I also express my gratitude to my fellow labmates, including Mr. V. Pandey, Mrs. N. Agarwal, Ms. S. Pandey, Ms. B. Diana, Ms. P. Singh, Ms. E. Taslima, Mrs. S. Shiwangi, Mr. S. Yadav, Mrs. N. Sen, and Dr. S. Kalra. Their contributions have significantly contributed to creating a wonderful and collaborative lab environment.

Lastly, but certainly not the least, I want to express my deepest gratitude to my parents, late Mr. Pyarelal Singh and Mrs. Rawana Devi, for their unwavering support and encouragement throughout this journey. Their love and guidance have been my guiding light, and I am forever indebted to them. Their belief in me has been a source of strength, especially during challenging times. Sadly, my father is no longer with us, but his memory and teachings continue to inspire me every day. I would also like to extend my thanks to my other family members, who have been a constant source of love and motivation. Their understanding and encouragement have been instrumental in keeping me focused and determined to complete this assignment successfully. Their presence has made this accomplishment even more special, and I am truly grateful for their support.

Date: 04/03/2024 Vijay Kumar Singh

Abstract

This thesis focuses on the development of prescribed-time adaptive and optimal control techniques for a class of nonlinear systems. The research addresses the challenges associated with uncertain nonlinear dynamics, constrained feedback control, achieving consensus in uncertain nonlinear multi-agent systems and nonlinear optimal feedback control. The ultimate goal is to achieve convergence within a specified time frame.

The thesis begins by introducing the concept of prescribed-time adaptive backstepping control. The stability and convergence properties of this control scheme are analyzed, and theoretical proofs and simulations are provided to validate its effectiveness.

Next, the prescribed-time adaptive backstepping control methodology is extended to address the control challenges in a twin rotor helicopter system. A detailed controller design is developed, taking into account the specific dynamics of the helicopter. Simulations and comparative analyses are conducted to evaluate the performance of the proposed control strategy.

This thesis further focuses on the development and analysis of a prescribed-time constrained feedback control scheme to handle parameter uncertainties and input constraints in real-world systems, with a specific application to a twin rotor helicopter exhibiting uncertain dynamics. The proposed control approach is designed to achieve convergence within a prespecified time frame while considering the challenges associated with uncertainties and input constraints. The work rigorously analyzes the stability of the proposed control approach to ensure its adaptability to parameter variations. To validate its performance, numerical simulations and practical experiments are conducted. The results demonstrate the control scheme's ability to successfully handle parameter uncertainties, maintain stability, and adhere to the input constraints in a real-world application.

The thesis also addresses the application of prescribed-time adaptive control to DC-DC boost converters, which are widely used in power electronic systems. An adaptive

estimator-based control strategy is proposed to regulate the output voltage of the boost converter within a prescribed time, considering uncertainties and variations in system parameters. The effectiveness of the proposed control approach is evaluated through theoretical analysis, simulation and experimental results.

Furthermore, the thesis investigates the achievement of prescribed-time adaptive neural consensus in uncertain nonlinear multi-agent systems. A novel consensus control approach is developed, combining adaptive control and neural networks, to achieve prescribed-time convergence and consensus among the agents. Theoretical analysis, simulation studies, and performance evaluations are provided to demonstrate the effectiveness of the proposed methodology.

Lastly, the thesis addresses the problem of prescribed-time optimal feedback control for nonlinear dynamical systems. Optimal nonlinear control techniques are employed to optimize a performance criterion subject to prescribed-time constraints. The theoretical developments, numerical methods, and experimental case studies presented in this thesis demonstrate the efficacy of the proposed prescribed-time optimal control methodology.

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Nomenclature

List of Greek and Roman Symbols

$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}_{\geq 0}$	Set of non-negative real numbers
$\mathbb{R}_{\geq k}$	Set of real numbers greater than or equal to k
$\mathbb{R} \to \mathbb{R}$	Element-wise mapping
$\mathcal{G}$	Undirected graph
$\mathcal{V}$	Vertex set
$\mathcal{E}$	Edge set
$\mathcal{A}$	Adjacency matrix of $\mathcal{G}$
$\mathcal{L}$	Graph laplacian of $\mathcal{G}$

Abbreviations

VSC	Variable Structure Control
SMC	Sliding Mode Control
PID	Proportional–Integral–Derivative
UAVs	Unmanned Aerial Vehicle
FPGAs	Field-Programmable Gate Arrays
GPUs	Graphics Processing Units
MIMO	Multi-Input Multi-Output
UBST	Upper Bound of Settling Time
MASs	Multi-Agent Systems
NNs	Neural Networks
RBF	Radial Basis Function
HJB	Hamilton–Jacobi–Bellman
ISS	Input to State Stable
ISMC	Integral Sliding Mode Control
min	Minimum
max	Maximum
sup	Supremum
s.t.	Such that