Chapter 1

Introduction

Control system theory forms the cornerstone of modern methods for regulating and managing dynamic systems across a vast range of applications. It encompasses a structured set of principles, methodologies, and tools dedicated to achieving system stability, desired performance, and robustness, even in the presence of disturbances, uncertainties, and varying environmental conditions. Fundamentally, control theory is concerned with the modeling, analysis, and design of systems that can autonomously track target behaviors or maintain prescribed outputs.

Spanning fields such as engineering, physics, economics, and the biological sciences, control theory provides a systematic framework for understanding and optimizing the behavior of dynamic systems. Central to this framework is the concept of feedback, where the system's output is continuously monitored and compared to a reference, with corrective actions applied to minimize any deviation. This feedback-driven approach allows systems to dynamically adapt to changing conditions and maintain stability.

Control systems are generally categorized into continuous-time and discrete-time types. While continuous-time systems deal with signals that evolve smoothly over time, discrete-time systems process signals sampled at specific intervals. Both types have found widespread applications in industries including electrical engineering, mechanical systems, aerospace, and process control.

At the heart of control system theory lies mathematical modeling, where system dynamics are typically described by differential equations for continuous-time systems and by difference equations for discrete-time systems. Stability analysis is crucial, assessing whether a system can return to its equilibrium after experiencing a disturbance. Furthermore, control design techniques are applied to shape the system's behavior, ensuring compliance with performance criteria while safeguarding resilience against uncertainties.

The tuning of control parameters is essential for tailoring system behavior to meet specified performance objectives. Various control strategies—such as Proportional-Integral-Derivative (PID) control, state-space methods, and optimal control—have been developed to enhance the precision, adaptability, and reliability of modern control systems.

Control methodologies are often classified into conventional and non-conventional approaches. Conventional control, typically grounded in the continuous dynamics framework, uses tools like Bode plots and Nyquist diagrams for stability and performance evaluation. PID controllers are prevalent here, providing robust performance in linear time-invariant systems, particularly in domains like process control and automotive engineering.

In contrast, non-conventional control strategies, such as variable structure control (VSC), feature dynamic switching and structural discontinuities to address system uncertainties more effectively. A prominent technique within VSC is Sliding Mode Control (SMC), where the system's state is driven onto a designated sliding surface and maintained there. The sliding motion grants the system remarkable robustness against model inaccuracies and external disturbances.

SMC is especially distinguished for its ability to handle nonlinearities and uncertainties. By forcing the system to operate on the sliding surface, it ensures that performance remains stable despite perturbations. Typically, SMC relies on discontinuous control laws that incorporate switching actions, enhancing resilience and guaranteeing stability. Formal stability proofs often utilize Lyapunov-based methods, providing rigorous guarantees of convergence even in uncertain settings.

The choice between conventional and variable structure control strategies depends heavily on system characteristics. Conventional methods are well-suited for precisely modeled, linear systems, while variable structure control techniques like SMC excel in handling nonlinear, time-varying, or highly uncertain systems—making them ideal for complex applications such as robotics, aerospace, and advanced manufacturing.

1.1 Motivation

- Nonlinear DTS often face disturbances and uncertainties, making precise control and stability a significant challenge. Moreover, the complete system model is not always available. Conventional control approaches typically depend on accurate system models; however, model inaccuracies and external disturbances can adversely affect their performance. In contrast, data-driven control (DDC) methods, which utilize input-output data through CFDL, offer a promising alternative by reducing or eliminating the reliance on precise system modeling.

- Controlling discrete-time nonlinear systems under PPC remains a significant challenge, particularly in the presence of uncertainties and disturbances. Traditional model-based methods often require precise system models and additional switching gains, complicating implementation and potentially degrading performance. To address these issues, our research proposes a minimum operator-based DDC strategy that eliminates reliance on explicit modeling and additional switching mechanisms. A FFDL approach constructs a data-driven model, while a transformative error mapping function converts constrained tracking errors into an unconstrained form, simplifying controller design. The resulting minimum operator-based discrete sliding mode control (MDSMC) ensures robust convergence within specified asymmetric bounds without introducing structural complexity.

- Uncertain systems pose significant challenges in control, especially when precise tracking and disturbance rejection are required under dynamic conditions. Traditional control methods often rely on full system state feedback, which may be infeasible due to the complexity of the system, measurement limitations, or the need for expensive sensors. Additionally, many conventional control strategies assume a constant, high sampling rate, which is not always practical in real-time systems with resource constraints. This can lead to reduced performance and increased implementation complexity. To address these challenges, this thesis introduces a novel approach that achieves quasi-sliding mode in uncertain systems using a fast output sampling control strategy combined with minimum operator-based DSMC. By operating solely on output samples, the proposed method eliminates the need for full system state feedback and reduces reliance on high sampling rates, making it well-suited for systems with limited resources.

- In the control of uncertain DTS, challenges arise due to the presence of disturbances, model inaccuracies, and the need for precise tracking under varying conditions. Traditional control strategies, while effective in many scenarios, often struggle with ensuring robustness when system uncertainties or disturbances are present. Furthermore, many approaches are prone to undesirable phenomena such as chattering during the sliding phase, which can degrade performance or lead to instability. To address these issues, there is a growing need for adaptive and robust control methods that can handle uncertainties without relying on fixed parameters or switching gains. Moreover, it is crucial that these methods offer flexibility in controlling the reaching time for the sliding surface, allowing the system to meet predefined performance specifications while avoiding chattering. Our research aims to fill these gaps by introducing an adaptive reaching law that guarantees the system reaches the sliding surface within a predetermined time while maintaining robust performance. The proposed method eliminates chattering effects and adapts control gains to the system's evolving behavior, thus ensuring effective control even in the presence of disturbances and uncertainties.

- Real-world applications such as two-tank systems, robotic manipulators, magnetic levitation systems, and helicopter systems highlight the critical requirement for advanced control techniques capable of handling uncertainties and guaranteeing predefined performance levels.

1.2 Literature Review

- **Data-Driven Discrete Sliding Mode Control:** In recent years, there has been increasing demand for higher control precision, particularly in managing nonlinear DTS affected by uncertainties and disturbances. SMC has been widely applied due to its robustness and ease of implementation, although its performance in discrete systems often suffers as sampling times increase. DSMC has seen significant advancements in two primary areas: the inclusion of switching and non-switching [2,4–7] elements in the RL. The concept of quasi-sliding mode (QSM) has been developed to address the lack of ideal sliding mode behavior in DTS. However, the transition of VSC from continuous-time systems (CTS) to DTS has not been entirely straightforward, often requiring a model-based approach that assumes an accurate understanding of the system dynamics, which is not always feasible [8–12]. DDC methods have emerged as an alternative to model-based control, providing flexibility in real-world applications where model accuracy is challenging. DDC methods generally fall into two categories: one involves adjusting parameters based on measured data while maintaining a fixed controller structure [13], and the other utilizes function approximations, such as neural networks or Taylor series, to fine-tune controller parameters [14], an approach known as model-free adaptive control (MFAC). The integration of MFAC with CFDL data-driven modeling, where controllers are designed using only input-output data without relying on system dynamics, has shown promise in handling nonlinearities in systems. The fusion of CFDL with DSMC techniques has been explored to address these nonlinearities effectively. Early work by [15] introduced model-free adaptive sliding mode control but did not account for disturbances. More recent studies [16] have proposed adaptive DSMC using CFDL for discrete nonlinear systems, improving disturbance handling. Researchers have also combined DSMC with MFAC to tackle industrial applications, including robotic joint speed control [17], pressure control systems [18], and MIMO systems [19]. The concept of discrete finite-time stability (FTS), initially not central to DSMC, has gained attention, especially following the introduction of a minimum-based approach [20]. This concept has been extended to high-order systems [9], achieving finite-time convergence. Further work has focused on reducing chattering and achieving ideal sliding mode in the absence of disturbances [2]. While combining MDSMC with CFDL has the potential to enhance performance, this approach remains unexplored in the literature. Our research proposes a novel integration of MDSMC with CFDL for nonlinear DTS with perturbations, offering a promising direction for future research.

- **Prescribed Performance-Based Data-Driven Control Using Minimum Operator:** In recent years, the increasing demand for precision control in nonlinear DTS has heightened the focus on addressing perturbations and uncertainties while ensuring PPC are met. Many practical systems require PP [21], ensuring that errors stay within predefined bounds. Although much of the work on PP control focuses on CTS [22], exploring this concept for DTS is crucial, given their widespread use in applications such as digital control and communication systems. The gap between continuous- and discrete-time control methods, exacerbated by sampling errors, complicates the direct application of CTS methods to DTS. While limited research on PP for DTS exists [23], a recent attempt to formulate MFAC using DSMC with features for PPC was made in [24], where the authors developed a data-driven adaptive sliding mode control (SMC) strategy based on the FFDL technique. Recently, minimum operator-based DSMC [2] has been developed. However, no prior attempt has been made to integrate the minimum operator-based DSMC approach with FFDL and PPC simultaneously, and the potential for performance improvement through this combination remains unexplored.

- **Output Feedback Based Discrete-Time Sliding Mode Control Using Minima Operator:** The SMC theory, developed by Utkin for CTS, adapts the controller's structure to achieve desired outcomes despite disturbances, parameter uncertainties, and performance constraints [25]- [26]. However, implementing SMC in DTS often leads to performance degradation as sampling time increases [27]. Research on DSMC has evolved, addressing methods for both full-state feedback [28], [29] and output feedback [30]- [31], though the former assumes access to all plant states, which is often impractical. Data-driven methods leveraging output feedback have gained traction, especially with advancements in digital computing [32]- [33], fostering research in DTS and DSMC [4], [5], [34], [35]. Recent research extends DSMC to multirate systems, where varying dynamics across channels and hardware constraints make multiple rates necessary. This approach has become crucial in networked control systems with bandwidth limitations [31], [36], [37]. MROF, which utilizes output readings and prior input signals, has proven effective for maintaining performance with lower sampling frequencies. In [36], FOS is introduced, offering a flexible method for system pole allocation while stabilizing controlled dynamics. Periodic intermittent control combined with MROF is employed in [30] for robust stabilization, while [31] presents a rapid sampling strategy to achieve QSM without control switching, minimizing chattering.

- **Adaptive Discrete-Time Sliding Mode Control:** SMC is a robust nonlinear control strategy, particularly effective for uncertain systems, offering advantages in terms of stability and resilience to external perturbations and uncertainties. It has found widespread applications in diverse fields, including robotics, aerospace, and motion control [38]. DSMC, in contrast to continuous sliding mode control (CSMC), operates with a limited switching frequency, making it more practical for real-world applications [39]. However, DSMC faces challenges such as chattering and lengthy reaching times. Chattering is a key issue that has prompted extensive research on switching functions and boundary layers [40–42]. Additionally, the reduction of reaching time is crucial, especially in applications like missile control that require stringent time constraints [43]. To address these challenges, several strategies have been proposed, including modified RLs that aim to reduce chattering and improve convergence, as seen in [44]. Despite these improvements, DSMC methods still face limitations, particularly the need for prior knowledge of disturbance bounds, which is not always available. To overcome this, adaptive sliding mode control, which adjusts without relying on disturbance knowledge, has become a promising alternative [45–47]. However, issues related to over- or under-estimation of the adaptive switching gain persist [45].

1.3 Contributions

- **Development of DDC control using Minimum Operator-Based CFDL-DSMC Design Approach:** A novel DDC control strategy combining CFDL with minimum operator-based DSMC has been proposed to enhance tracking precision and disturbance rejection.

- **Development of DDC control using Minimum Operator-Based FFDL-DSMC Design Approach with prescribe performance feature:** A FFDL with minimum operator based DDC method has been introduced to achieve tracking within specified asymmetric ranges, utilizing a transformative error mapping technique that converts constrained errors into unconstrained forms without requiring additional switching gains.

- **Output Feedback Based Discrete-Time Sliding Mode Control Using Minima Operator:** A quasi-sliding mode control strategy based on fast output sampling and minimum operators has been developed, requiring only output data for controller design and achieving finite-time control.

- **Adaptive Minimum Operator DSMC:** An adaptive control law with minimum operator and ceiling function-based gain adjustment has been proposed, allowing predetermined reaching time and mitigating chattering during sliding motion.

- **Finite-Time Attitude Tracking in Helicopter Systems:** A discrete finite-time backstepping control approach using minimum operators has been implemented and validated experimentally for a two-DOF helicopter system, ensuring robust finite-time tracking with settling time estimation.

- **Finite-Time Control of Magnetic Levitation System:** A discrete-time minimum operator-based sliding mode controller has been designed and validated for a magnetic levitation system, achieving improved stabilization and reducing chattering compared to conventional approaches.

1.4 Organization of the Thesis

This thesis is organized into eight chapters:

- **Chapter 1** introduces the work, providing a detailed literature survey, identifying the research gap, and outlining the contributions of the thesis.

- **Chapter 2** presents the notation and preliminaries used throughout the development of the thesis.

- **Chapter 3** discusses the methodology, focusing on the development of the DDC algorithms based on CFDL and minimum operator-based techniques.

- **Chapter 4** presents the methodology, focusing on the development of the DDC algorithms based on FFDL and minimum operator-based techniques with prescribed performance features.

- **Chapter 5** presents the development of the MROF-based DSMC control using minimum operators.
- **Chapter 6** provides an adaptive switching scheme design using a ceiling function for discrete-time sliding mode control.
- **Chapter 7** provides finite-time discrete control for a two-DOF helicopter system.
- **Chapter 8** presents a discrete-time sliding mode controller for a magnetic levitation system using minimum operators.
- **Chapter 9** concludes the thesis and suggests potential future research directions.