Abstract

This thesis advances the field of continuous-time optimization by developing dynamical systems that guarantee convergence to optimal solutions within a predefined time. We propose novel approaches to solve time-invariant, time-varying, unconstrained, and constrained optimization problems with a predefined-time convergence property, where the upper bound of settling time can be selected a priori. Lyapunov based convergence analysis is discussed for the proposed dynamical systems. Furthermore, we exploit gradient flows to infer passivity properties of physical systems, facilitating controller design.

We begin by presenting a modified gradient flow (GF) with a predefined time convergence to solve unconstrained optimization problems with strongly convex objective functions. The application of the modified GF to solve unconstrained optimization problems is also examined under the Polyak–Łojasiewicz condition, which serves as a relaxation of the strong convexity assumption. Lyapunov-based analysis and numerical simulations illustrate the effectiveness of the proposed methods.

We then integrate the idea of predefined-time convergent gradient flow dynamics and projected gradient techniques to solve convex optimization problems with linear equality constraints. We propose a projected gradient flow dynamics with a predefined-time convergence property. A perturbed projected gradient flow is also studied within the framework of input-to-state stability. Examples based on linear least squares and resource allocation problems are solved using the proposed algorithm.

Next, we extended the idea of predefined time gradient flows to solve time-varying optimization problems. Optimization problems emerging in most of the real-world applications are dynamic, where either the objective function or the constraints change continuously over time. We present dynamical systems for tracking the optimal trajectory of time-varying optimization problems with time-varying constraints in a predefined time. A robust Newton-like approach is developed for cases, where the exact knowledge of the rate of change of the gradient of objective function is unknown. Levenberg-Marquardt-like dynamics is proposed for the cases when the Hessian of the objective function is singular or near singular. Lyapunov based convergence analysis is discussed for the proposed methods. We demonstrate the applicability of the proposed method through its use in a collision-free robot navigation problem.

Further, the thesis also investigates the application of predefined time gradient flows in calculating passivity indices, which is a quantitative measure of the passivity properties of a system. Passivity indices are useful in controller design. We propose a predefined time gradient flow dynamics to calculate passivity indices which is formulated as an optimal solution to an optimization problem. With the help of predefined time gradient flows, we analyze the structure and system properties from data first and leverage this knowledge for controller design.

Finally, using the calculated passivity indices via gradient flows, we design controllers for discrete-time linear time invariant systems. In addition, we exploit the feedback interconnection property of passive systems to achieve asymptotic as well as finite-time stability of the discrete-time system utilizing only passivity indices. We validate the proposed method's effectiveness through application to a coupled tank system.