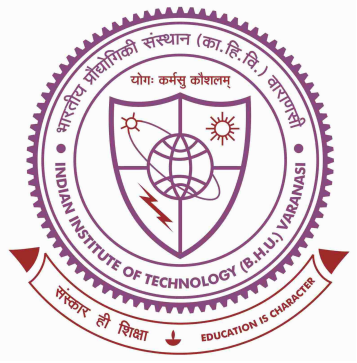
Sliding Mode based on Difference Equation with Minima



Thesis submitted in partial fulfillment
for the award of degree

Doctor of Philosophy

by

Parijat Prasun

DEPARTMENT OF ELECTRICAL ENGINEERING
Indian Institute of Technology
(Banaras Hindu University)
Varanasi

Roll No: 19081001

2023

To my family, whose patience, understanding, and encouragement sustained me during the countless hours spent in the pursuit of knowledge. Your belief in my abilities fueled my determination to overcome challenges and persist in the face of complexity.

CERTIFICATE

It is certified that the work contained in the thesis titled **Sliding Mode based on Difference Equation with Minima** by **Parijat Prasun** has been carried out under my supervision and this work has not been submitted elsewhere for a degree. It is further certified that the student has fulfilled all the requirements of the Comprehensive Examination, Candidacy, and State-of-the-Art Seminar for the award of Ph.D. Degree.

Supervisor

Dr. Shyam Kamal

Dept. of Electrical Engg.

IIT(BHU) Varanasi

Varanasi - 221005

Co-Supervisor

Dr. Sandip Ghosh

Dept. of Electrical Engg.

IIT(BHU) Varanasi

Varanasi - 221005

DECLARATION

I, **Parijat Prasun**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. Shyam Kamal** and **Dr. Sandip Ghosh** from July 2019 to December 2023, at the Departement of Electrical Engineering, Indian Institute of Technology (BHU) Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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Parijat Prasun

(Parijat Prasun)

CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my/our knowledge.

Dr. Shyam Kamal

IIT(BHU) Varanasi

Dr. Sandip Ghosh

IIT(BHU) Varanasi

10/10/24

Signature of Head of Department/Coordinator of School

प्रोफेसर एवं विभागाध्यक्ष / PROFESSOR & HEAD
विद्युतीय अभियांत्रिकी विभाग / Department of Electrical Engineering
भारतीय प्रौद्योगिकी संस्थान / Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय) / (Banaras Hindu University)
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Parijat Prasun

Date: 10/10/2024

Parijat Prasun

List of Figures

Nomenclature

List of Greek and Roman Symbols

$\mathbb{R}$	Set of real numbers
$\mathbb{R}_+$	Set of positive real numbers
$\mathbb{R}^n$	n-dimensional vector space over the real number field $\mathbb{R}$
$\mathbb{R}_{\geq 0}$	Set of non-negative real numbers
$\mathbb{Z}_{\geq 0}$	Set of non-negative integers
$\mathbb{R}_{\geq k}$	Set of real numbers greater than or equal to k
$\mathbb{R} \to \mathbb{R}$	Element-wise mapping
$\mathcal{K}$	Class $\mathcal{K}$ function
$\mathcal{K}_\infty$	Class $\mathcal{K}_\infty$ function
$\mathcal{L}$	Class $\mathcal{L}$ function
$\mathcal{KL}$	Class $\mathcal{KL}$ function
$\mathcal{GK}$	Generalized-$\mathcal{K}$ function
$\mathcal{GKL}$	Generalized-$\mathcal{KL}$ function

Abbreviations

VSC	Variable Structure Control
SMC	Sliding Mode Control
CSMC	Continuous-Time Sliding Mode Control
DSMC	Discrete-Time Sliding Mode Control/Discrete Sliding Mode Control
FTS	Finite-Time Stable
ISS	Input-to-State Stable
FTISS	Finite-Time Input-to-State Stable
MTBP	Matched Type Bounded Perturbations
QSM	Quasi-Sliding Mode
HOSM	Higher-Order Sliding Mode
STA	Super-Twisitng Algorithm
STO	Super-Twisitng Observer
DEM	Difference Equation with Minima
CTS	Continuous-Time System
DTS	Discrete-Time System
PID	Proportional Integral and Derivative
ZOH	Zero-Order Hold
RL	Reaching Law
min	Minimum
max	Maximum
s.t.	Such that