8 Anisotropic Cosmological Models with Bulk Viscosity and Particle Creation in Saez-Ballester Theory of Gravitation

8.1 Introduction

The aim of modern cosmology is to study the past history, the present state and future evolution of the universe. In recent years, there has been a lot of interests in alternative theories of gravitation (Brans and Dicke (1961), Canuto et al. (1977), Saez and Ballester (1985)). Particle creation processes are supposed to play important role in the early evolution of the universe. Phenomenologically, particle creation has been described in terms of effective bulk viscosity coefficients (Zel'dovich (1970), Hu (1982), Lima and Germano (1992), Zimdahl and Pavon (1993)). Prigogine et al. (1988) have introduced a phenomenological model of particle creation by considering thermodynamics of open system in the framework of cosmology. They have suggested that matter creation takes place out of gravitational energy in irreversible

The content of this chapter has been accepted for publication in Pramana Journal of Physics (2015)

process of non-equilibrium dynamics. This type of creation basically corresponds to irreversible energy flow from gravitational field to the created matter constituent. The rate of particle production can also be described by the quantum field theory in curved space-time (Birrell and Davies (1982)). Parker (1968) has suggested the idea of particle creation in the expanding universe. Hoyle and Narlikar (1962) have suggested the idea of continuous creation of matter which was subsequently modified in the form of quasi-steady-state cosmology (Hoyle, Burbidge and Narlikar (1994)).

Bulk viscosity is another irreversible process which contributes to entropy production in the universe. It has been suggested that dissipative process in the early stages of the expansion of the universe may well account for the present degree of isotropy and the ratio of number of photons to baryons. Eckart (1940) developed the first relativistic theory of non-equilibrium thermodynamics to study the effect of bulk viscosity. Most of the works with bulk viscosity have been studied in standard Eckart's theory of irreversible processes. However, it was pointed out later that the Eckart's theory has serious drawbacks concerning casuality and stability.

Several authors have explored the idea of cosmological models with bulk viscosity and particle creation.The evolution of Bianchi cosmologies with bulk viscosity and particle creation has been studied by Krori and Mukherjee (2000). The effect of bulk viscosity on the evolution of FRW models is studied by Desikan (1997). Johri and Pandey (1999) discussed a new class of FRW models with matter creation and analyzed behaviors of corresponding models . Singh and Beesham (1999) and Singh et al.(2002) studied cosmological models having bulk viscosity and particle creation within the framework of Brans-Dicke theory. Gr⊘n (1990) and Maartens (1995) presented exhaustive reviews on cosmological models with non-causal and causal thermodynamics. Singh and Kale (2011) studied anisotropic bulk viscous cosmological models with particle creation in general relativity. They have also studied the role of particle creation and bulk viscosity in the evolution of homogeneous and anisotropic model of the universe represented by Bianchi type I metric in Brans-Dicke theory (Singh and Kale (2011)). Chaubey (2012) has found the solution for Bianchi type V bulk viscous cosmological models with particle creation in Brans-Dicke theory of gravitation.

In this chapter, we study particle creation and bulk viscosity in spatially homogeneous and anisotropic Bianchi type-V metric in Saez-Ballester theory of gravitation. In Sec.(8.2), we present the basic equations. In Sec.(8.3), we present field equations in Saez-Ballester theory of gravitation. In Sec.(8.4), we obtain exact solutions of the field equations by applying a special law of variation of Hubble's parameter that yields the constant value of the deceleration parameter. Two types of cosmological models are presented, one with power-law expansion and the other one with exponential expansion. The bulk viscosity coefficient is calculated for Full Causal, Eckart's and truncated theories in both models. We summarize the chapter in Sec.(8.5).

8.2 Basic Equations

The effective energy- momentum tensor T_{ij} of the standard Einstein's field equations in the presence of particle creation and bulk viscosity, which includes the creation pressure term p_c and the bulk viscous stress Π, is given as

$$T_{ij} = (\rho + p + p_c + \Pi)\, u_i u_j - (p + p_c + \Pi)\, g_{ij} \tag{8.1}$$

where ρ is energy density, p the pressure and u^i is the four velocity vector of the fluid satisfying $u^i u_i = 1$.

The particle number density flow vector $N^i (= \eta u^i)$ and the entropy flux vector S^i $(= \nu \eta u^i)$ in second law of thermodynamics suggest the following equations:

$$N^i_{;i} = \dot{\eta} + 3\eta H = \Gamma, \tag{8.2}$$

$$S^i_{;i} = \eta\dot{\nu} + \nu\Gamma \geq 0 \tag{8.3}$$

where η is particle number density, ν is entropy per particle, H is Hubble parameter and Γ is the source term which will be positive if there is production of particles and it is negative when there is annihilation of particles. In the absence of particle creation or annihilation, it is zero.

The Gibbs equation for an open thermodynamical system may be written as

$$\eta T \dot{\nu} = \dot{\rho} - (\rho + p)\frac{\dot{\eta}}{\eta} \tag{8.4}$$

where T is the temperature of the cosmic fluid. Here the expression for entropy per particle is given by

$$\dot{\nu} = -\frac{3Hp_c}{\eta T} - \frac{3H\Pi}{\eta T} - \frac{(\rho + p)}{\eta^2 T}\Gamma. \tag{8.5}$$

For an open adiabatic system in cosmology, the supplementary pressure p_c, due to the creation of matter, assumes the following form (Prigogine et al. (1988)):

$$p_c = -\frac{(\rho + p)}{3H\eta}\Gamma = -\frac{(\rho + p)}{3H}\left(3H + \frac{\dot{\eta}}{\eta}\right). \tag{8.6}$$

Equation (8.6) predicts the amount of pressure arising from particle creation. From Eqs.(8.5) and (8.6), the expression for entropy per particle is given by

$$\dot{\nu} = -\frac{3H\Pi}{\eta T}. \tag{8.7}$$

From Eqs.(8.4) and (8.7), we obtain

$$\frac{\dot{\eta}}{\eta} = \frac{\dot{\rho} + 3H\Pi}{\rho + p}. \tag{8.8}$$

Using the equation of state $p = \gamma\rho(0 \leq \gamma \leq 1)$ in Eq.(8.8), the particle number density can be obtained as

$$\eta^{1+\gamma} = \eta_0 \rho \exp\left(\int 3H\Pi\rho^{-1}dt\right) \tag{8.9}$$

where η_0 is an integration constant. The conventional bulk viscous effect in Bianchi universe can be modeled within the framework of non-equilibrium thermodynamics. The transport equation for the bulk viscous pressure Π takes the form (Maartens (1995)):

$$\Pi + \tau\dot{\Pi} = -3\xi H - \frac{\epsilon\tau\Pi}{2}\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right) \tag{8.10}$$

where ξ is the bulk viscous coefficient and τ is the relaxation time associated with the dissipative effect.

We study the behavior of bulk viscosity in **Full Causal theory** ($\epsilon = 1$), **Eckart's theory** ($\tau = 0$) and **Truncated theory** ($\epsilon = 0$). In truncated theory the above evolution equation (8.10) reduces to

$$\Pi + \tau\dot{\Pi} = -3\xi H. \tag{8.11}$$

In this theory, to ensure that the viscous signals do not exceed the speed of light, we consider the following relation

$$\tau = \frac{\xi}{\rho}. \tag{8.12}$$

In Full Causal theory, the equation of state for pressure and temperature (Singh and Kale (2011), Arbab (1997)) are taken to be barotropic i.e., $p = \gamma\rho$ and $T = T(\rho)$. Then

$$T \propto \exp\left(\int \frac{dp(\rho)}{\rho + p(\rho)}\right). \tag{8.13}$$

This will reduce to the following equation

$$T = T_0 \rho^{\frac{\gamma}{(1+\gamma)}} \tag{8.14}$$

where T_0 is a constant. Using Eqs.(8.12) and (8.14), the evolution equation (8.10) reduces to

$$\Pi + \frac{\xi}{\rho}\dot{\Pi} = -3H\xi - \frac{\xi\Pi}{2\rho}\left[3H - \frac{(1+2\gamma)\dot{\rho}}{(1+\gamma)\rho}\right]. \tag{8.15}$$

In Eckart's non-causal theory, the evolution equation (8.10) will reduce to

$$\Pi = -3\xi H. \tag{8.16}$$

8.3 Field Equations

We now solve the field equations for the anisotropic Bianchi type V metric in the presence of particle creation and bulk viscous fluid within the framework of Saez-Ballester (1985) theory of gravitation.

In comoving coordinate system, the field equation (1.17) and (1.18) for the metric (2.1) lead to the following equations:

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{m^2}{A^2}=-(p+p_c+\Pi)+\frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{8.17}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{m^2}{A^2}=-(p+p_c+\Pi)+\frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{8.18}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{m^2}{A^2}=-(p+p_c+\Pi)+\frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{8.19}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{3m^2}{A^2}=\rho-\frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{8.20}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0, \tag{8.21}$$

$$\frac{\ddot{\phi}}{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{r}{2\phi}\dot{\phi}^2=0. \tag{8.22}$$

By combining Eqs.(8.17)-(8.22), we obtain the continuity equation:

$$\dot{\rho}+3(\rho+p)H=-3(p_c+\Pi)H. \tag{8.23}$$

We now obtain exact solutions of the field equations (8.17)-(8.22). Integrating Eq.(8.21) and observing the integration constant into function B or C, we obtain

$$A^2=BC. \tag{8.24}$$

Following the procedure of Saha and Rikhvitsky (2006) on Eqs.(8.17)-(8.19) and (8.24), the metric functions can explicitly be written in terms of the average scale factor

$$A(t)=a, \tag{8.25}$$

$$B(t)=B_0 a\exp\left(\frac{X}{3}\int\frac{dt}{a^3}\right), \tag{8.26}$$

$$C(t) = {B_0}^{-1} a \exp\left(-\frac{X}{3}\int\frac{dt}{a^3}\right) \tag{8.27}$$

where B_0 and X are integration constants.

We can find the metric functions A, B and C from Eqs.(8.25)-(8.27),if the expression of the average scale factor a is known. In order to find a, we assume that the Hubble parameter H is related to the average scale factor a by

$$H = la^{-n} \tag{8.28}$$

where $l > 0$ and $n \geq 0$ are constants. This type of relation has already been considered by Berman (1983), Berman and Gomide (1988) for solving field equations in FRW models. Such a relation gives a constant value of the deceleration parameter. From Eqs.(1.32) and (8.28), we obtain

$$\dot{a} = la^{-n+1}, \tag{8.29}$$

$$\ddot{a} = -l^2(n-1)a^{-2n+1}. \tag{8.30}$$

From Eqs.(1.34), (8.29) and (8.30), we find that

$$q = n - 1. \tag{8.31}$$

We see that under the law of variation of H in Eq.(8.28), the deceleration parameter q is constant. From Eq.(8.29), we obtain the expressions for the average scale factor as

$$a = (nlt)^{\frac{1}{n}}, n \neq 0, \tag{8.32}$$

$$a = a_0 \exp(lt), n = 0 \tag{8.33}$$

where a_0 and l are constants of integration.

8.4 Solutions of Field Equations

We derive cosmological models corresponding to the cosmic scale functions A, B and C by using the power-law and exponential forms of the average scale factor a given in Eqs.(8.32) and (8.33) separately.

8.4.1 Model I

Using Eq.(8.32) into Eqs.(8.25)-(8.27) and integrating, we obtain

$$A = (nlt)^{1/n}, \tag{8.34}$$

$$B = B_0(nlt)^{1/n} \exp\left[M(nlt)^{\frac{n-3}{n}}\right], \tag{8.35}$$

$$C = {B_0}^{-1}(nlt)^{1/n} \exp\left[-M(nlt)^{\frac{n-3}{n}}\right] \tag{8.36}$$

where the constant M is given by

$$M = \frac{X}{l(n-3)}, n \neq 3. \tag{8.37}$$

Equations (8.22) has the general solution

$$\phi = \phi_0(nlt)^{\frac{2(n-3)}{n(r+2)}} \tag{8.38}$$

where $\phi_0 = \left[\frac{h(r+2)}{2l(n-3)}\right]^{\frac{2}{r+2}}$ is a constant.

The expansion scalar, shear scalar, directional Hubble's parameter, generalized Hubble parameter, the volume scalar and anisotropic parameter are given by

$$\theta = \theta_0(nlt)^{-1}, \tag{8.39}$$

$$\sigma^2 = {\sigma_0}^2(nlt)^{-6/n}, \tag{8.40}$$

$$H_x = l(nlt)^{-1}, \tag{8.41}$$

$$H_y = l(nlt)^{-1} + N(nlt)^{-\frac{3}{n}}, \tag{8.42}$$

$$H_z = l(nlt)^{-1} - N(nlt)^{-\frac{3}{n}}, \tag{8.43}$$

$$H = l(nlt)^{-1}, \tag{8.44}$$

$$V = (nlt)^{3/n}, \tag{8.45}$$

$$A_m = A_0(nlt)^{\frac{2(n-3)}{n}} \tag{8.46}$$

where θ_0, σ_0, N and A_0 are constants.

From Eq.(8.20), the value of the energy density can be found as

$$\rho = \rho_1(nlt)^{-2} - \rho_2(nlt)^{-6/n} - \rho_3(nlt)^{-2/n} \tag{8.47}$$

where ρ_1, ρ_2, ρ_3 are constants.

If we assume the baratropic equation of state $p = \gamma\rho$, expression for the pressure is

$$p = \gamma\left[\rho_1(nlt)^{-2} - \rho_2(nlt)^{-6/n} - \rho_3(nlt)^{-2/n}\right]. \tag{8.48}$$

We observe that the scale factors A, B and C are zero at the initial time $t = 0$. The expansion scalar, shear scalar, Hubble parameter and the three directional Hubble's parameter are all infinite at $t = 0$. The spatial volume is zero at this epoch. The mean anisotropy parameter is infinite at $t = 0$ for $n < 3$. The energy density and pressure tends to infinite at this epoch. All these values of different physical parameters show that the universe starts evolving with zero volume and expand with cosmic time. Thus, the model has a point singularity at $t = 0$. The spatial volume tends to infinite at $t \to \infty$. The expansion scalar, shear scalar, energy density, pressure and mean anisotropic parameter will all become zero for large time. These indicate that the universe is expanding with increase of cosmic time but the rate of expansion and shear scalar decrease to zero and finally tend to isotropic. This model approaches isotropic during late time of its evolution as $\lim \frac{\sigma}{\theta} = 0$ for $t \to \infty$.

Now in the following subsections, we study the behavior of particle creation and bulk viscosity of this model in four different physical laws.

8.4.1.1 Bulk Viscosity Energy-Density Law

We assume that the bulk viscous stress Π is associated with energy density ρ by the following relationship

$$\Pi = \Pi_0 \rho^\alpha \tag{8.49}$$

where Π_0 is a constant. This relationship is motivated by the relation

$$\xi = \xi_0 \rho^\alpha \tag{8.50}$$

where $\xi_0 \geq 0$, $\alpha \geq 0$ (Weinberg (1968), Murphy, (1973)) assumed $\alpha = 1$ in the case of small density. The case $\alpha = 1$ corresponds to a radiative fluid.

The expressions for bulk viscous stress Π and creation pressure p_c can be written in the forms

$$\Pi = \Pi_0 \left[\rho_1 (nlt)^{-2} - \rho_2 (nlt)^{-6/n} - \rho_3 (nlt)^{-2/n}\right]^\alpha, \tag{8.51}$$

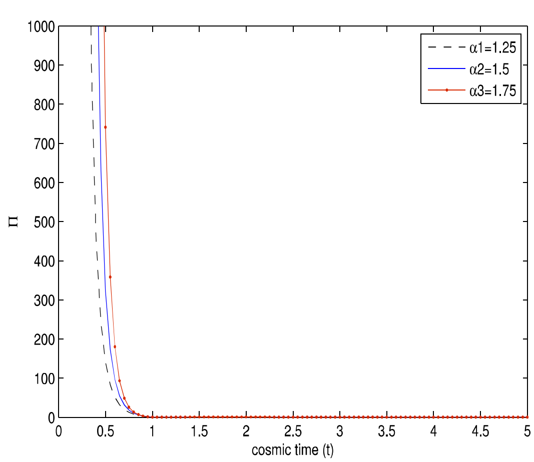
$$p_c = F_1(t) - \Pi_0 \rho^\alpha \tag{8.52}$$

where

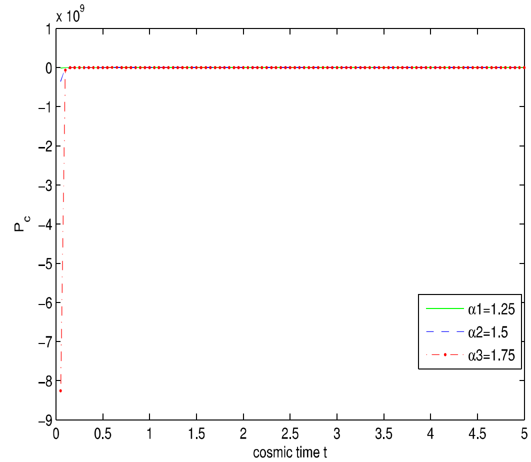
$$F_1(t) = p_1 (nlt)^{-2} + p_2 (nlt)^{\frac{-6}{n}} + p_3 (nlt)^{\frac{-2}{n}}, \tag{8.53}$$

p_1, p_2, p_3 being constants.

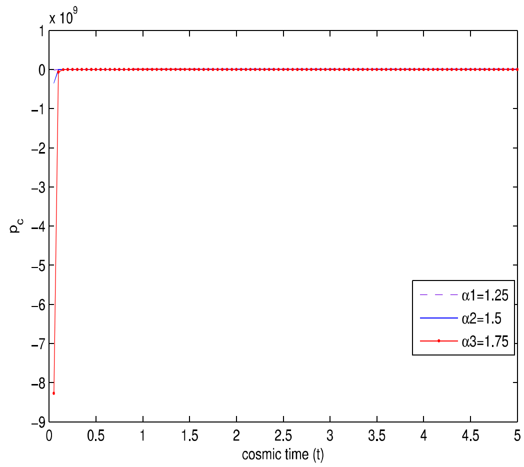
The graphical behaviors of Π and p_c, given in Eq.(8.51) and Eq.(8.52), are shown in the fig. (8.1).



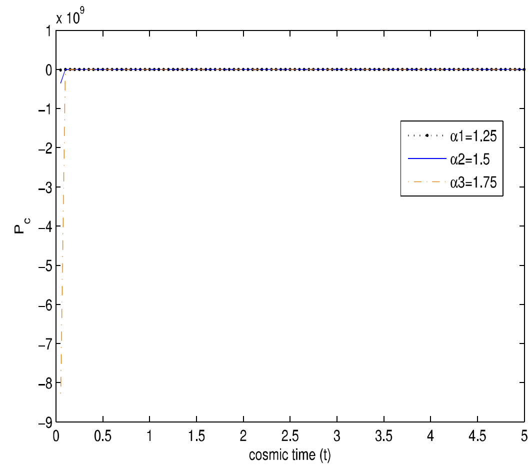
(a)



(b)



(c)



(d)

Figure 8.1: Panel a indicates the variation of bulk viscous stress vs. time t for different values of α and $n = 1.5$ and panel b, c and d represent particle creation pressure for different values of α and for $\gamma = 0$,$\gamma = 1$ and $\gamma = 1/3$ and n $= 1.5$ respectively.

Now, the bulk viscosity coefficient in different theories are given as follows:

Full Causal theory:

$$\xi = \rho^{\alpha+2}[F_2(t)\rho^2 + F_3(t) + F_4(t)\rho^{\alpha+1} + F_5(t)\rho^{\alpha-1}]^{-1} \tag{8.54}$$

where $F_2(t) = \xi_1(nlt)^{-1}$, $F_3(t) = \xi_2(nlt)^{-3} - \xi_3(nlt)^{-\frac{6}{n}-1} - \xi_4(nlt)^{-\frac{2}{n}-1}$, $F_4(t) = \xi_5(nlt)^{-1}$, $F_5(t) = \xi_6(nlt)^{-3} + \xi_7(nlt)^{-\frac{6}{n}-1} + \xi_8(nlt)^{-\frac{2}{n}-1}$, $\xi_1, ..., \xi_8$ being constants.

Eckart theory:

$$\xi = F_6(t)\rho^{\alpha} \tag{8.55}$$

where $F_6(t) = \xi_9 t$, ξ_9 being constant.

Truncated theory:

$$\xi = F_7(t) + F_8(t)\rho^{\alpha-1} \tag{8.56}$$

where $F_7(t) = \xi_{10}(nlt)^{-3} - \xi_{11}(nlt)^{-\frac{6}{n}-1} - \xi_{12}(nlt)^{-\frac{2}{n}-1}$, $F_8(t) = \xi_{13}(nlt)^{-\frac{6}{n}-1} + \xi_{14}(nlt)^{-\frac{2}{n}-1} - \xi_{15}(nlt)^{-3}$, $\xi_{10}, ..., \xi_{15}$ being constants.

8.4.1.2 Uniform Particle Number Density ($\dot{\eta} = 0$)

We consider the case when particle number density is uniform during evolution of the universe. This assumption leads to the particle production term Γ and creation pressure p_c as

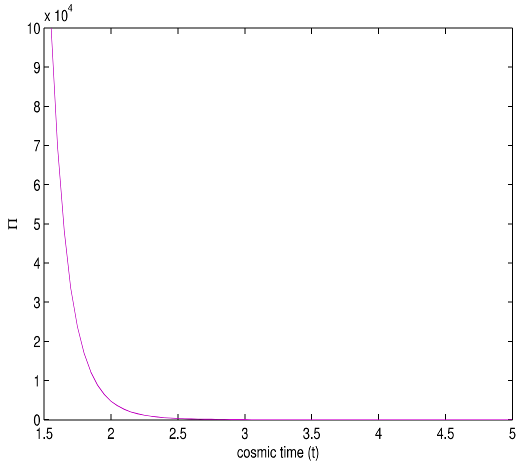
$$\Gamma = 3H\eta, \tag{8.57}$$

$$p_c = -(1+\gamma)\rho. \tag{8.58}$$

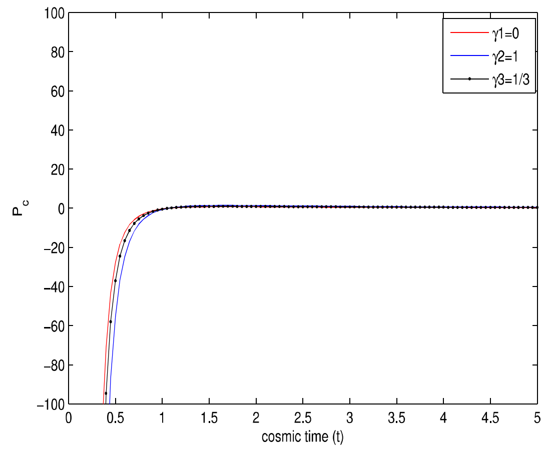
The value of bulk viscous stress can be obtained as

$$\Pi = \Pi_1(nlt)^{-2} - \Pi_2(nlt)^{-6/n} - \Pi_3(nlt)^{-2/n} \tag{8.59}$$

where Π_1, Π_2 and Π_3 are constants. The graphical behavior of bulk viscous stress Π and particle creation pressure p_c can be observed through the fig. (8.2).



(a)



(b)

Figure 8.2: Panel a represents variation of Π with time t for $n = 1.5$ and b represents variation of p_c with time for $\gamma = 0$, $\gamma = 1$, $\gamma = \frac{1}{3}$ and $n = 1.5$ respectively.

The coefficient of bulk viscosity in all three cases takes the following form :

Full Causal theory:

$$\xi = -\Pi\rho^2[F_9(t)\rho^2 + F_{10}(t)\rho - F_{11}(t)\Pi]^{-1} \tag{8.60}$$

where $F_9(t) = 3l(nlt)^{-1}$, $F_{10}(t) = \xi_{16}(nlt)^{-3}+\xi_{17}(nlt)^{-\frac{6}{n}-1}+\xi_{18}(nlt)^{-\frac{2}{n}-1}$, $F_{11}(t) = \xi_{19}(nlt)^{-3} + \xi_{20}(nlt)^{-\frac{6}{n}-1} + \xi_{21}(nlt)^{-\frac{2}{n}-1}$, $\xi_{16}, ..., \xi_{21}$ being constants.

Eckart theory:

$$\xi = F_{12}(t) \tag{8.61}$$

where $F_{12}(t) = \xi_{22}(nlt)^{-\frac{6}{n}+1}+\xi_{23}(nlt)^{-\frac{2}{n}+1}-\xi_{24}(nlt)^{-1}$, $\xi_{22}, ..., \xi_{24}$ being constants.

Truncated theory:

$$\xi = -\Pi\rho[F_{13}(t)]^{-1} \tag{8.62}$$

where $F_{13}(t) = \xi_{25}(nlt)^{-3} + \xi_{26}(nlt)^{-\frac{6}{n}} - \xi_{27}(nlt)^{-\frac{2}{n}}$, $\xi_{25}, ..., \xi_{27}$ being constants.

8.4.1.3 Ideal Gas

In the case of an ideal gas $\Gamma = 0$ and $p_c = 0$. Then Eq.(8.2) becomes

$$\dot{\eta} + 3\eta H = 0. \tag{8.63}$$

Equation (8.63) on integration gives

$$\eta = \eta_1 t^{\frac{-3}{n}} \tag{8.64}$$

where η_1 is an integrating constant. Equation (8.63) is the expression for particle creation density. This model has only bulk viscosity and bulk viscous stress is obtained as

$$\Pi = \Pi_4 (nlt)^{-2} + \Pi_5 (nlt)^{-6/n} + \Pi_6 (nlt)^{-2/n} \tag{8.65}$$

where Π_4, Π_5 and Π_6 are constants. Fig.(8.3) shows the graphical behavior of bulk viscous stress Π for Ideal Gas in model I.

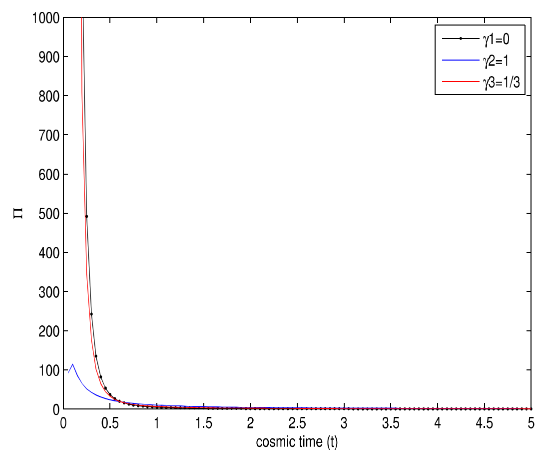


Figure 8.3: Variation of Π with time for $\gamma = 0$, $\gamma = 1$, $\gamma = \frac{1}{3}$ and $n = 1.5$

Here the expression for bulk viscous coefficient assumes the following forms:

Full Causal theory

$$\xi = F_{14}(t)\rho^2[F_{15}(t)\rho + F_{16}(t)\rho^2 + F_{17}(t)F_{18}(t)]^{-1} \tag{8.66}$$

where $F_{14}(t) = \xi_{28}(nlt)^{-2} + \xi_{29}(nlt)^{-\frac{6}{n}} - \xi_{30}(nlt)^{-\frac{2}{n}}$, $F_{15}(t) = \xi_{31}(nlt)^{-3} + \xi_{32}(nlt)^{-\frac{6}{n}-1} - \xi_{33}(nlt)^{-\frac{2}{n}-1}$, $F_{16}(t) = 3l(nlt)^{-1}$, $F_{17}(t) = \xi_{34}(nlt)^{-2} + \xi_{35}(nlt)^{-\frac{6}{n}} + \xi_{36}(nlt)^{-\frac{2}{n}}$, $F_{18}(t) = \xi_{37}(nlt)^{-3} - \xi_{38}(nlt)^{-\frac{2}{n}-1} - \xi_{39}(nlt)^{-\frac{6}{n}-1}$, $\xi_{28}, ...,\xi_{39}$ being constants.

Eckart theory:

$$\xi = F_{19}(t) \tag{8.67}$$

where $F_{19}(t) = \xi_{40}(nlt)^{-1} + \xi_{41}(nlt)^{-\frac{6}{n}+1} - \xi_{42}(nlt)^{-\frac{2}{n}+1}$, $\xi_{40}, ..., \xi_{42}$ being constants.

Truncated theory :

$$\xi = F_{20}(t)\rho[F_{21}(t)]^{-1} \tag{8.68}$$

where $F_{20}(t) = \xi_{43}(nlt)^{-2} + \xi_{44}(nlt)^{-\frac{6}{n}} - \xi_{45}(nlt)^{-\frac{2}{n}}$, $F_{21}(t) = \xi_{46}(nlt)^{-3} + \xi_{47}(nlt)^{-\frac{6}{n}-1} - \xi_{48}(nlt)^{-\frac{2}{n}-1}$, $\xi_{43}, ...,\xi_{48}$ being constants.

8.4.1.4 Creation with Second Order Correction in H

Triginer and Pavon (1994) suggested a generalization of the conservation of total particle number in standard cosmology by considering the Taylor expansion of $\frac{\dot{\eta}}{\eta} = f(H)$ upto second order in H as

$$\frac{\dot{\eta}}{\eta} = -3H + dH^2 \tag{8.69}$$

where d is a constant. The particle number density η also satisfies the balance equation

$$\dot{\eta} + 3\eta H = \Gamma. \tag{8.70}$$

So, from above equation

$$\Gamma = d\eta H^2. \tag{8.71}$$

Equation (8.71) suggests that for $d > 0$, $d = 0$ and $d < 0$, respectively, there is creation, no creation and annihilation of particles. Using eq.(8.69), the expression for creation pressure is as follows:

$$p_c = -\frac{(1+\gamma)d}{3} H\rho. \tag{8.72}$$

Hence the expression for creation pressure and bulk viscous stress can be obtained as

$$p_c = F_{22}(t) \tag{8.73}$$

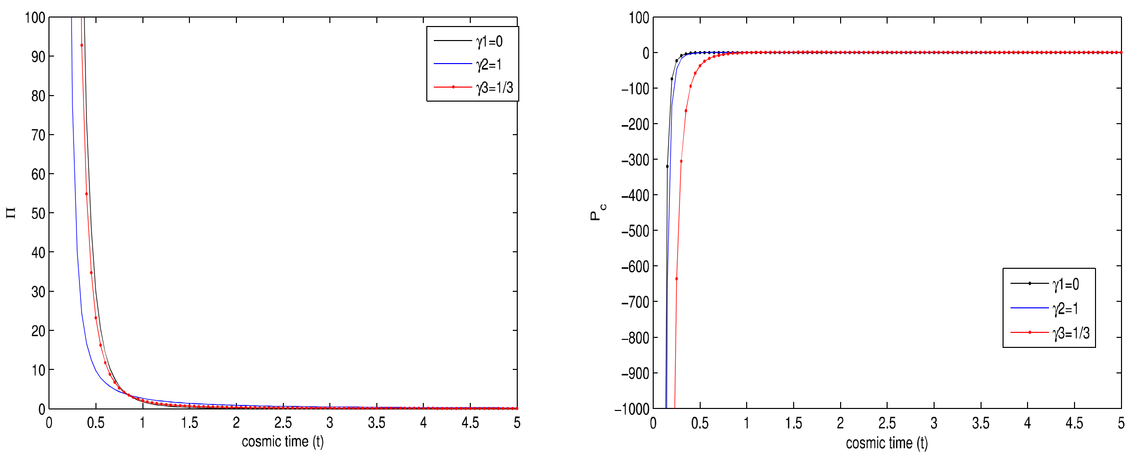
where

$$F_{22} = p_4(nlt)^{\frac{-6}{n}-1} + p_5(nlt)^{\frac{-2}{n}-1} - p_6(nlt)^{-3}, \tag{8.74}$$

p_4, p_5 and p_6 being constants and

$$\begin{aligned} \Pi &= \Pi_7(nlt)^{-2} + \Pi_8(nlt)^{-3} + \Pi_9(nlt)^{\frac{-2}{n}} - \Pi_{10}(nlt)^{\frac{-2}{n}-1} \\ &\quad -\Pi_{11}(nlt)^{\frac{-6}{n}} - \Pi_{12}(nlt)^{\frac{-6}{n}-1} \end{aligned} \tag{8.75}$$

where $\Pi_7, ...\Pi_{12}$ being constants.



(a) (b)

Figure 8.4: Panel a and b represent variation of Π and p_c with time t for $\gamma = 0$, $\gamma = 1$, $\gamma = \frac{1}{3}$ and $n = 1.5$ respectively.

Fig.(8.4) shows the behavior of Π and p_c respectively in different parameters.

The expressions for coefficient for bulk viscosity in different theories are given as:

Full Causal theory:

$$\xi = F_{23}(t)\rho^2[F_{24}(t)\rho + F_{25}(t)\rho^2 + F_{26}(t)F_{27}(t)]^{-1} \tag{8.76}$$

where $F_{23}(t) = -\xi_{49}(nlt)^{-2} - \xi_{50}(nlt)^{-3} - \xi_{51}(nlt)^{-\frac{2}{n}} + \xi_{52}(nlt)^{-\frac{2}{n}-1} + \xi_{53}(nlt)^{\frac{-6}{n}} + \xi_{54}(nlt)^{-\frac{6}{n}-1}$, $F_{24}(t) = \xi_{55}(nlt)^{-3} - \xi_{56}(nlt)^{-4} - \xi_{57}(nlt)^{-\frac{2}{n}-1} + \xi_{58}(nlt)^{-\frac{2}{n}-2} + \xi_{59}(nlt)^{-\frac{6}{n}-1} + \xi_{60}(nlt)^{-\frac{6}{n}-2}$, $F_{25}(t) = 3l(nlt)^{-1}$, $F_{26}(t) = \xi_{61}(nlt)^{-2} + \xi_{62}(nlt)^{-3} + \xi_{63}(nlt)^{-\frac{2}{n}} - \xi_{64}(nlt)^{-\frac{2}{n}-1} - \xi_{65}(nlt)^{-\frac{6}{n}} - \xi_{66}(nlt)^{-\frac{6}{n}-1}$, $F_{27}(t) = \xi_{67}(nlt)^{-3} - \xi_{68}(nlt)^{-\frac{2}{n}-1} - \xi_{69}(nlt)^{-\frac{6}{n}-1}$, $\xi_{49}, ...,\xi_{69}$ being constants.

Eckart theory:

$$\xi = F_{28}(t) \tag{8.77}$$

where $F_{28}(t) = -\xi_{70}(nlt)^{-1} - \xi_{71}(nlt)^{-2} + \xi_{72}(nlt)^{-\frac{2}{n}} - \xi_{73}(nlt)^{-\frac{2}{n}+1} + \xi_{74}(nlt)^{-\frac{6}{n}} + \xi_{75}(nlt)^{-\frac{6}{n}+1}$, $\xi_{70}, ..., \xi_{75}$ being constants.

Truncated theory:

$$\xi = F_{29}(t)\rho[F_{30}(t)]^{-1} \tag{8.78}$$

where $F_{29}(t) = -\xi_{76}(nlt)^{-2} - \xi_{77}(nlt)^{-3} - \xi_{78}(nlt)^{-\frac{2}{n}} + \xi_{79}(nlt)^{-\frac{2}{n}-1} + \xi_{80}(nlt)^{-\frac{6}{n}} + \xi_{81}(nlt)^{-\frac{6}{n}-1}$, $F_{30}(t) = \xi_{82}(nlt)^{-3} - \xi_{83}(nlt)^{-4} - \xi_{84}(nlt)^{-\frac{2}{n}-1} + \xi_{85}(nlt)^{-\frac{2}{n}-2} + \xi_{86}(nlt)^{-\frac{6}{n}-1} + \xi_{87}(nlt)^{-\frac{6}{n}-2}$, $\xi_{76}, ...,\xi_{87}$ being constants.

8.4.2 Model II

We derive in this section an exponentially expanding cosmological model with particle creation and bulk viscosity. Substituting the value of a given by Eq.(8.33) into

Eqs.(8.25)-(8.27) and integrating, we obtain

$$A = a_0 \exp{(lt)}, \tag{8.79}$$

$$B = B_1 a_0 \exp{[lt - P \exp{(-lt)}]}, \tag{8.80}$$

$$C = {B_1}^{-1} a_0 \exp{[lt + P \exp{(-lt)}]} \tag{8.81}$$

where B_1 and P are constants. Using this value of a, the general solution of Eq.(8.22) is given by

$$\phi = \phi_1 \exp{(-\phi_2 t)} \tag{8.82}$$

where

$$\phi_1 = \left\{ \frac{h(r+2)}{-6la_0^3} \right\}^{\frac{2}{r+2}},$$
$$\phi_2 = \frac{6l}{r+2}$$

and h are constants.

For the cosmological model with scale factors given by Eqs.(8.79)-(8.81), the dynamical parameters are given by

$$\theta = 3l, \tag{8.83}$$

$$\sigma^2 = {\sigma_1}^2 \exp(-2lt), \tag{8.84}$$

$$V = V_0 \exp(3lt), \tag{8.85}$$

$$H = l, \tag{8.86}$$

$$H_x = l, \tag{8.87}$$

$$H_y = l + Q \exp{(-lt)}, \tag{8.88}$$

$$H_z = l - Q \exp{(-lt)}, \tag{8.89}$$

$$A_m = A_1 \exp(-2lt) \tag{8.90}$$

where σ_1, V_0, Q and A_1 are constants.

If we assume the equation of state $p = \gamma\rho$, then energy density and pressure are calculated as

$$\rho = \rho_4 - \rho_5 \exp(-2lt), \tag{8.91}$$

$$p = \gamma\left[\rho_4 - \rho_5 \exp(-2lt)\right] \tag{8.92}$$

where ρ_4 and ρ_5 are constants.

We observe that all the three scale factors A, B, C the spatial volume V, expansion scalar θ, shear scalar σ, anisotropy parameter A_m, the energy density ρ are all constants at the time $t = 0$. Therefore the cosmological model has no singularity at $t = 0$. These results show that the universe starts evolving with constant volume and expands with uniform exponential expansion and volume grows exponentially with time. We see that all the dynamical parameters are decreasing functions of time and ultimately tends to zero as $t \to \infty$. The mean anisotropy parameter tends to zero for large time. Since $q = -1$, the universe is accelerating . Since $\lim \frac{\sigma}{\theta} = 0$ for $t \to \infty$, the model tends to isotropy for large time.

In this case we also study in the following subsections the behavior of particle creation and bulk viscosity of this model in four different laws.

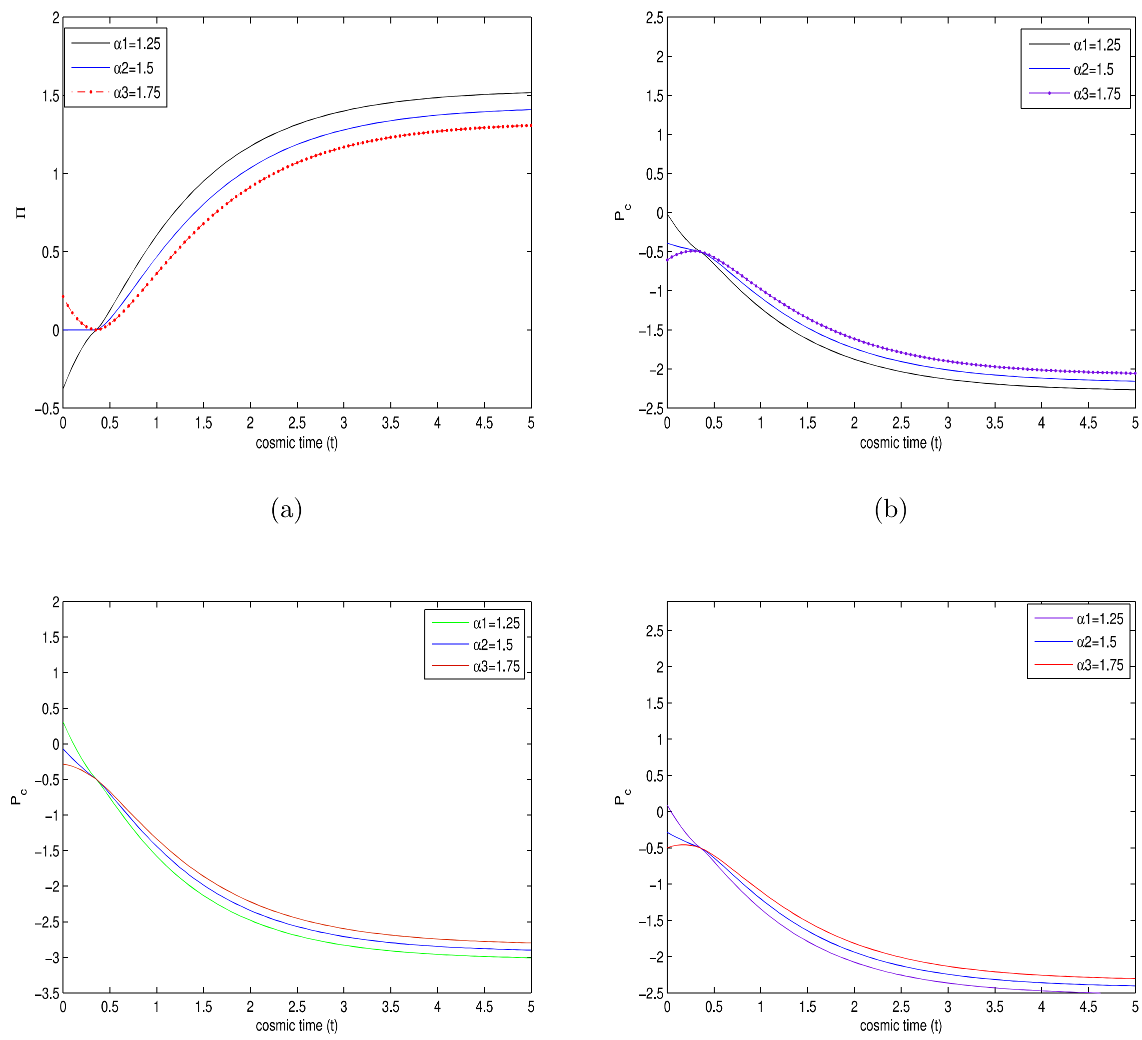
8.4.2.1 Bulk Viscosity Energy-Density Law

In this case, again, a relation between viscous pressure and energy density is considered as in Eq.(8.49). Then the expression for bulk viscous stress and creation pressure can be obtained as

$$\Pi = \Pi_0[\rho_4 - \rho_5 \exp(-2lt)]^{\alpha}. \tag{8.93}$$

$$p_c = G_1(t) - \Pi_0 \rho^{\alpha}. \tag{8.94}$$

From fig.(8.5), we can observe the behavior of Π and p_c respectively.



(a) (b) (c) (d)

Figure 8.5: Panel a indicates the variation of bulk viscous stress vs. time t for different values of α and $n = 1.5$ and panel b, c and d represent particle creation pressure for different values of α and for $\gamma = 0$, $\gamma = 1$ and $\gamma = 1/3$ respectively.

The expression for bulk viscosity coefficient in three theories can be obtained as:

Full Causal theory

$$\xi = -2\Pi_0 \rho^{\alpha+2}[G_2(t)\rho^{\alpha} + G_3(t)\rho^{\alpha+1} + G_4(t)\rho^2 - G_5]^{-1} \tag{8.95}$$

where $G_2(t) = \xi_{88}\exp(-2lt)$, $G_3(t) = 3\Pi_0 l_0 = constant$, $G_4(t) = 6l = constant$, $G_5(t) = \xi_{89}\exp(-2lt)$, ξ_{88}, ξ_{89} being constants.

Eckart theory:

$$\xi = -\xi_{90}\rho^{\alpha} \tag{8.96}$$

where ξ_{90} is constant.

Truncated theory

$$\xi = -\Pi_0 \rho^{\alpha+1}[G_6(t) + G_7(t)\rho^{\alpha-1}]^{-1} \tag{8.97}$$

where $G_6(t) = \xi_{91} - \xi_{92}\exp(-2lt)$, $G_7(t) = \xi_{93}\exp(-2lt)$, $\xi_{91}, ..., \xi_{93}$ being constants.

8.4.2.2 Uniform Particle Number Density ($\dot{\eta} = 0$)

Considering uniform particle number density ($\dot{\eta} = 0$), the expressions for the bulk viscous stress and creation pressure are obtained as

$$\Pi = -\Pi_{13}\exp(-2lt), \tag{8.98}$$

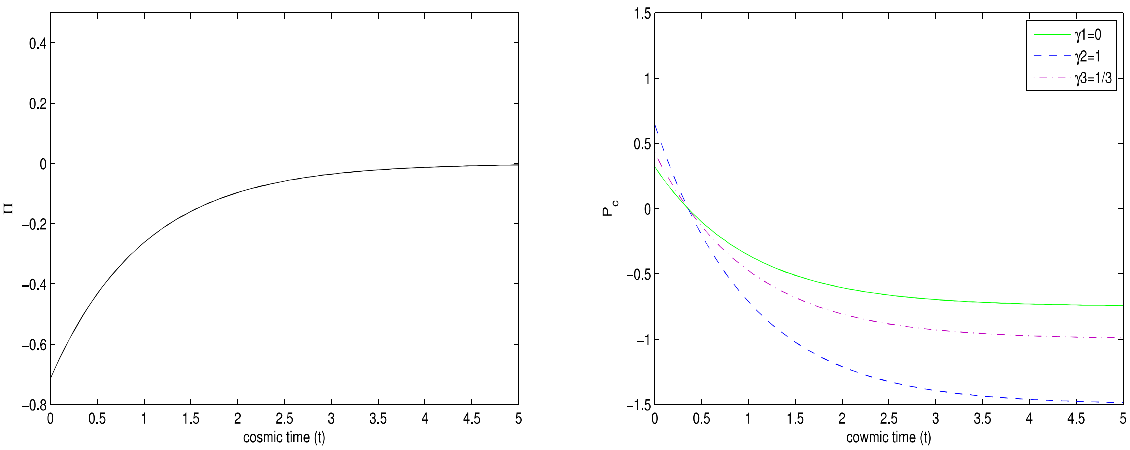
$$p_c = -(1+\gamma)\rho. \tag{8.99}$$

The graphical behavior of bulk viscous stress Π and particle creation pressure p_c can be observed through the fig. (8.6).

The expression for bulk viscosity coefficient in three theories can be obtained as:

Full Causal theory

$$\xi = G_8(t)\rho^2[G_9(t)]^{-1} \tag{8.100}$$



(a) (b)

Figure 8.6: Panel a represents variation of Π with time t for $n = 1.5$ and b represents variation of p_c with time t for $\gamma = 0$, $\gamma = 1$, $\gamma = \frac{1}{3}$ and $n = 1.5$ respectively.

where $G_8(t) = \xi_{94}\exp(-2lt)$, $G_9(t) = \xi_{95}\exp(-2lt) + \xi_{96}\exp(-4lt)$, $\xi_{94}, ...\xi_{96}$ being constants.

Eckart theory:

$$\xi = G_{10}(t) \tag{8.101}$$

where $G_{10}(t) = \xi_{97}\exp(-2lt)$, ξ_{97} is constant.

Truncated theory:

$$\xi = G_{11}(t)\rho[G_{12}(t)]^{-1} \tag{8.102}$$

where $G_{11}(t) = \xi_{98}\exp(-2lt)$, $G_{12}(t) = \xi_{99} - \xi_{100}\exp(-2lt)$, $\xi_{98}, ..., \xi_{100}$ being constants.

8.4.2.3 Ideal Gas

The value of particle number density and bulk viscous stress in this case can be obtained as

$$\eta = \eta_2 \exp(-3lt), \tag{8.103}$$

$$\Pi = -\Pi_{14} + \Pi_{15} \exp(-2lt) \tag{8.104}$$

where η_2, Π_{14} and Π_{15} are constants. Nature of Π in this case can be observed in fig.(8.7).

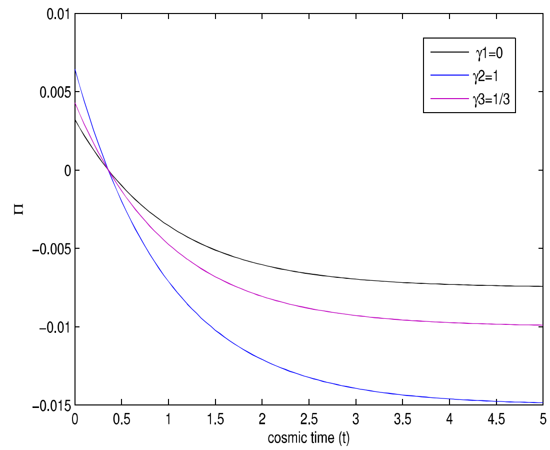


Figure 8.7: Variation of Π with time t for $\gamma = 0$, $\gamma = 1$ and $\gamma = 1/3$

Here, the bulk viscosity coefficient assumes the following forms:

Full Causal theory:

$$\xi = G_{13}(t)\rho^2[G_{14}(t)]^{-1} \tag{8.105}$$

where $G_{13}(t) = \xi_{101} - \xi_{102} \exp(-2lt)$, $G_{14}(t) = \xi_{103} + \xi_{104} \exp(-2lt) - \xi_{105} \exp(-4lt)$, $\xi_{101}, ...\xi_{105}$ being constants.

Eckart theory:

$$\xi = G_{15}(t) \tag{8.106}$$

where $G_{15}(t) = \xi_{106} - \xi_{107} \exp(-2lt)$, ξ_{106}, ξ_{107} being constants.

Truncated theory

$$\xi = G_{16}(t)\rho[G_{17}(t)]^{-1} \tag{8.107}$$

where $G_{16}(t) = \xi_{108} - \xi_{109}\exp(-2lt)$, $G_{17}(t) = \xi_{110} - \xi_{111}\exp(-2lt)$, $\xi_{108}, ..., \xi_{111}$ being constants.

8.4.2.4 Creation with Second Order Correction in H

Also in this case, the expression for creation pressure and bulk viscous stress can be obtained as

$$p_c = G_{18}(t) \tag{8.108}$$

where

$$G_{18}(t) = -p_9 + p_{10}\exp(-2lt)$$

p_9 and p_{10} are constants.

$$\Pi = \Pi_{16} - \Pi_{17}\exp(-2lt) \tag{8.109}$$

where Π_{16} and Π_{17} are constants.

Fig.(8.8) shows the behavior of Π and p_c respectively in different parameters.

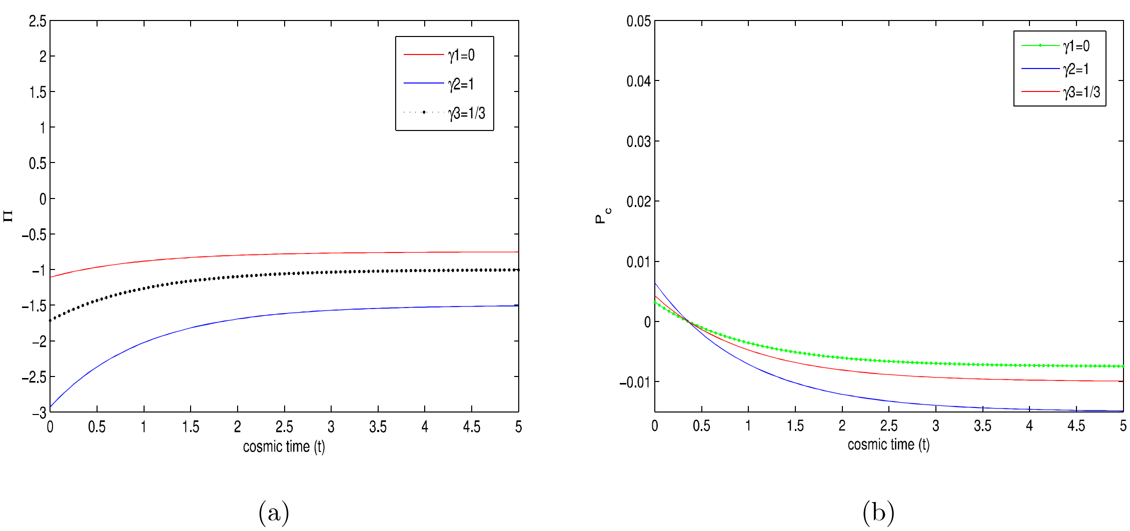


Figure 8.8: Panel a and b represent variation of Π and p_c with time t respectively for $\gamma = 0$, $\gamma = 1$, $\gamma = \frac{1}{3}$ and $n = 1.5$

Now the expressions for bulk viscous coefficient in all three cases can be written as:

Full Causal theory:

$$\xi = G_{19}(t)\rho^2[G_{20}(t)]^{-1} \tag{8.110}$$

where $G_{19}(t) = -\xi_{112}+\xi_{113}\exp(-2lt)$, $G_{20}(t) = \xi_{114}+\xi_{115}\exp(-2lt)-\xi_{116}\exp(-4lt)$, $\xi_{112}, ...\xi_{116}$ being constants.

Eckart theory:

$$\xi = G_{21}(t) \tag{8.111}$$

where $G_{21}(t) = -\xi_{117} + \xi_{118}\exp(-2lt)$, ξ_{117}, ξ_{118} being constants.

Truncated theory:

$$\xi = G_{22}(t)\rho[G_{23}(t) + G_{24}(t)\rho]^{-1} \tag{8.112}$$

where $G_{19}(t) = -\xi_{119} + \xi_{120}\exp(-2lt)$, $G_{20}(t) = \xi_{121}\exp(-2lt)$, $G_{21}(t) = 3l = constant$, $\xi_{119}, ..., \xi_{121}$ being constants.

8.5 Conclusions

In this chapter, we have considered field equations for a spatially homogeneous and anisotropic Bianchi type V space in the presence of bulk viscous with particle creation within the framework of Saez-Ballester theory of gravitation. Two types of solutions of the average scale factor, one is of power law type and other of exponential form, are obtained by using the special law of variation of Hubble's parameter that yields a constant value of deceleration parameter. We have presented models in two type of cosmologies which correspond to singular and non singular cosmological models. We have studied separately the bulk viscosity and particle creation in each model for four different cases. The bulk viscosity coefficient is obtained for full Causal , Eckart and truncated theories in all cases. We have also discussed physical and dynamical properties of both models.