Chapter 6

Conclusion and future work

6.1 Conclusion

The criticality of crack formation and its subsequent propagation in the context of solid mechanics cannot be overstated. This intense scrutiny is justified by cracks' pivotal role in compromising structural integrity. The field of fracture mechanics focuses on quantitatively describing the mechanical state of a deformed body that contains cracks. This highlights the need for a more comprehensive understanding of the material's flow characteristics to predict failure accurately. The thesis systematically explores multiple aspects of fracture mechanics, drawing upon the principles of elasticity and thermo-elasticity. Different chapters have addressed problems with insulated and conducting finite crack models, moving semi-infinite cracks under thermal and mechanical loadings. The thesis initiates its discourse by grounding the basics of fracture mechanics, presenting a detailed literature survey and fundamentals of elasticity theory through Chapter 1. This chapter is organized into sections and subsections, presenting the essential mathematical preliminaries, definitions, and theorems for the readership's comprehension. It further enriches this foundational understanding by providing a concise historical context and an overview of the discipline of Fracture Mechanics, referencing the seminal contributions within this domain. The dissertation further articulates the diverse methodologies employed across its chapters, comprehensively elaborating on their analytical and numerical dimensions.

Chapter 2 provides a mathematical framework for analyzing the problem with cracks under uniform heat flux. The geometry of the problem includes a central crack of length $2a$, and another crack is present at a distance e and height h from the origin. The chapter specifically addresses the interaction between cracks, considering their various length-to-height ratios. The main aim of the chapter is to find the expressions of mode-I SIFs at the vicinity of each crack's tip. The most important part of the study is the presentation of increasing and decreasing tendencies of the propagation of offset cracks due to variations in the lengths and distance of the cracks through the pictorial presentations of SIFs. With the help of the first kind of Chebysev polynomial, the expansion collocation method was employed to perform the numerical computations. Leveraging the first kind of Chebyshev polynomial, the expansion collocation method was adopted for the execution of the numerical analyses. The study opens avenues for extending the inquiry into the interplay between cracks in an orthotropic sandwich strip sandwiched between two planes, specifically with offset cracks, under the influence of thermo-mechanical loadings.

Chapter 3 addresses the crack problem in an anti-plane coordinate system. An intricate analysis of an anti-plane stress problem characterized by the presence of three distinct cracks embedded in dissimilar orthotropic media is undertaken. The mixed boundary value problem is converted into a set of dual integral equations. Subsequently, these equations are resolved by employing the Schmidt method, a reputable numerical strategy renowned for its efficacy in solving integral equations. The discourse extends to Jacobi polynomials, which are highlighted as a generalization of Chebyshev polynomials and recognized for forming an orthogonal set. The investigative study delves into three specific material combinations viz., Aluminium, Graphite epoxy, and Epoxy, which hold significance across various engineering domains due to their prevalent utilization. The examination is pivotal for enhancing the materials' serviceability and lifespan by elucidating their fracture mechanics. A noteworthy achievement of the present work is the meticulous determination of the variations in the SIF of mode III across the cracks situated in these composite materials. To facilitate a comprehensive understanding of the interactions of the cracks within the considered model, the plotted graphs are depicting the normalized SIFs at all crack tips against the ratio of crack lengths. These graphical representations, adjusted for varying widths of the horizontal strip signified by h, bestow profound insights into crack behaviour. This comprehensive scrutiny not only significantly advances the field of fracture mechanics but also facilitates the conceptualization and fabrication of more resilient and enduring composite materials for diverse engineering implementations.

In Chapter 4, the investigation focuses on a dynamically propagating semi-infinite crack located at the interface of two distinct strips. The classical Wiener-Hopf (W-H) method has been applied for the complex-valued transformed plane to solve the problem of unknown function in the half-plane. This scholarly endeavour has successfully met five distinct objectives through its rigorous analysis. The first one is the appropriate use of the W-H method to handle the W-H and the W-H dual equations by splitting those into two parts using a contour in the complex plane. Secondly, it elucidates the derivation of an asymptotic expression for SIF at the tip of the semi-infinite crack situated at the interface of two dissimilar orthotropic media.

The third one is finding the analytical expression of the normalized crack energy and then showcasing the pictorial presentations of the absolute crack energy for different crack velocities and depths of the composite. The fourth one is finding the exact length of the meeting of the two faces of the crack before its tip. This contribution is also the first of its kind for interfacial semi-infinite crack. Lastly, it ambitiously demonstrates, through the application of stress magnification factors (SMFs), the graphical representation of the influence wielded by the external strips over the composite strips harbouring the semi-infinite crack at their interfaces. This graphical representation is meticulously tailored to encompass a variety of composite strip depths and crack velocities, catering to specific case studies. This comprehensive investigation significantly advances the understanding of the dynamics and mechanics involved in the behaviour of semi-infinite cracks at the interfaces of dissimilar orthotropic media, laying down a solid foundation for future research endeavours in this domain.

Partially insulated cracks in the orthotropic medium under the influence of thermal and mechanical loadings have been studied in Chapter 5. The Schmidt method has been used for numerical computations, and the results reveal the SIFs for mode-I and mode-II, as well as HFIF and crack displacements for various cases. Numerical validations and comparisons have been carried out with previous known results. The expressions were computed semi-analytically and plotted numerically for different particular cases. The detailed analysis and computation of expressions for various physical parameters viz., HFIF, SIFs for mode-I and mode-II cracks, and crack displacements have been determined by semi-analytical and numerical techniques for the considered problem. The chapter also displayed the graphical comparisons and validation carried out to facilitate an analysis of the results obtained in the study with the existing results.

6.2 Limitations and Future works

The thesis has considered mathematical problems concerning the cracks in the orthotropic medium under the influence of thermal and mechanical loadings. Thermo-elasticity theory is an advancement to elasticity theory, which considers the effects of thermal and mechanical fields in an elastic body. For this purpose, Fourier's law has been applied to governing equations and problem formulations. Fourier's law of heat conduction is the basis of the classical theory of thermo-elasticity. With the rapid progress in aerospace engineering, the importance of thermal stresses is observed. As mentioned in the Chapter 2, Duhamel and Neumann, in the early *19-th* century, proposed the theory of coupling between thermal and mechanical fields and obtained the expressions for strain with thermal gradients. Later on, Thomson [152], Biot [153], Chadwick [154], and many others pioneered the theory of thermo-elasticity. The constitutive relations and basic governing equations of coupled thermo-elasticity have been discussed in detail in Chapter 1. All these theories are based on Fourier's law as stated in equation (2.2) and have been used in Chapters 2 and 4 of the thesis. Although the Fourier law is suitable for studying heat conduction in many engineering problems, it has presented unconvincing results on a small temperature and time scale [155]. Considering the limitations of Fourier's law in mathematics and physical aspects, in future works the aim would be to extend the thermo-mechanical stress problems considered in the thesis to non-Fourier theories. The motivation behind this is the advancement in microscale technologies and the development of thermo-elasticity theories, which consider the new constitutive variables and phase parameters of time and space. The future works will involve the non-Fourier generalized thermo-elastic theories applied to multiple crack problems. For this purpose, some generalized thermo-elasticity theories have been discussed.

Lord-Shulman thermo-elasticity theory [156]:

The heat conduction equation is given as

$$k\ T_{,jj} = (1 + \tau_q\ \frac{\partial}{\partial t})(\rho\ c\ \dot{T} + T_o\ \dot{u}_{j,j} - \rho\ R), \tag{6.1}$$

where k is conductivity, τ_q is thermal relaxation time, T is temperature, ρ is density, T_o is reference temperature, R is heat generation vector and c is wave velocity.

Green-Naghdi Theory:

The heat conduction equation for an anisotropic homogenous medium is given as [157]

$$\rho\ c\ \ddot{T} + \beta\ T_o\ \nabla \cdot \ddot{u} = k^*\ \nabla^2 T + k\ \nabla^2 \ddot{T} + R, \tag{6.2}$$

where k^* is the material property.

Dual-phase lag theory [158]: It is introduced by including two phase-lags in the Fourier heat conduction as

$$k(1 + \tau_T\ \frac{\partial}{\partial t})\nabla^2 T = (1 + \tau_q\ \frac{\partial}{\partial t})(\rho\ c\ \dot{T} + T_o\ \dot{u}_{j,j} - \rho\ R), \tag{6.3}$$

where τ_T is phase-lag of temperature gradient and τ_q is phase-lag of heat flux.

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