NUMERICAL STUDY OF CONSOLIDATION BEHAVIOUR FOR SATURATED AND UNSATURATED SOILS

A Thesis

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Doctor of Philosophy

By

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Under the supervision of

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ABSTRACT

The mathematical model representing the classical Terzaghi's (1925) one-dimensional consolidation theory was developed on the premises of a few drastic idealizations. With the advancement in computational flexibilities and vast researches on soil mechanics, most of the involved assumptions in the conventional approach of settlement estimation appear inconsequential. The present thesis revisits Terzaghi's (1925) one-dimensional consolidation theory, from the following few perspectives: (a) initial condition associated with the load-imposed excess pore pressure, (b) boundary condition connected to the drainage boundaries, (c) governing differential equations related to the loading pattern, velocity-gradient relationship, stress-induced heterogeneities, and saturation states, and (d) coupling of the stress-deformation alongwith the pore pressure dissipation equation. The classical Terzaghi's one-dimensional linear parabolic diffusion equation is sufficiently modified and used for performing the consolidation analysis of saturated soil. The entire numerical analyses are conducted by using the Crank-Nicolson finite difference scheme

An attempt has been made to study the consolidation of homogeneous and two-layered soil by considering ten different spatial distributions of initial excess pore water pressure (u_0) profiles. The variation of normalized pore pressure isochrones and the trend of the consolidating curves for different distributions of u_0 are extensively evaluated and critically analyzed. A new approach has been developed to study the consolidation phenomenon through the locus of the maximum point of the isochrones.

Next, a study is conducted to understand the consequences of stress-dependent permeability and compressibility on the consolidation of inherently homogenous clays. The

importance of stress- and spatial-gradient of permeability in the consolidation analysis is established. Furthermore, permeability variation is integrated with non-Darcian flow models under ramp loading. The spatial and temporal permeability variation (*k*-isochrones) are also plotted to explain the variation in the consolidation curves.

In contrast to the traditional uncoupled consolidation, coupled methods, which integrate pore water pressure dissipation with soil deformation, provide more reliable solutions, especially for time-dependent loading and other complicated conditions. A detailed one-dimensional coupled consolidation is performed to investigate the combined influence of semipermeable drainage boundaries, non-Darcian fluid flow, and periodic loading. The non-Darcian flow is realized by employing the exponential and threshold gradient-based non-linear model. Traditionally, to simplify the numerical scheme, the hydraulic gradient in the exponential term is evaluated at the previous (known) time state, which converts the nonlinear equation into a linear equation. By adopting Newton-Raphson technique, an effort has been initiated to solve the actual nonlinear equation by computing the gradients at the current (unknown) time state.

The construction of geotechnical structures frequently occurs under partially saturated conditions. The unsaturated consolidation is governed by Fredlund and Rahardjo (1993)'s classical diffusion equations. The present thesis re-evaluates the impact of initial pore pressure condition, drainage boundaries, and velocity-gradient relations on the transient flow. To provide flexibility in the drainage, mixed drainage boundary conditions for the air and water phases are applied at the top and bottom boundary surfaces. A series of numerical experimentations are carried out. The results offer valuable insights for geotechnical engineering applications in various environmental conditions.

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LIST OF NOTATIONS

Symbol	Definition
η	Ratio of loading time period and rest time period
σ	Total stress
σ'	Effective stress
σ_0'	Initial effective stress within the soil
θ	Model parameter for non-Darcian flow equation
γ_w	Unit weight of water
Δt	Time interval
α	Degree of skewness
χ	Angle of arched distributions
β_{aw}	Compressibility of the air-water mixture
β_w	Compressibility of the water
β_a	Compressibility of the air
β	Variable dependent on C_c and M
A	Void ratio where effective stress is 1kPa
a	Model parameter for non-Darcian flow equation
b_1, b_2,b_3 and b_4	Geometrical parameter of initial excess pore water pressure distribution
B	Void ratio intercept where permeability is 1m/sec
c_v	Coefficient of consolidation.
C_c	Compression index
c_v^w	Coefficient of consolidation for water phase in unsaturated soil
c_v^a	Coefficient of consolidation for air phase in unsaturated soil
D_u	Range of elastic deformation in U_{avg} curve
D_s	Range of elastic deformation in settlement curve
D_a	Stress-state independent transmissivity coefficient of the air phase.
e	Void ratio of soil
e_0	Initial void ratio of soil
E'	Exponential hydraulic gradient equation

E	Differentiated form of non-darcian velocity
h_2	The thickness of the bottom layer in two layerd soil strata
h_1	The thickness of the top layer in two layerd soil strata
H	Thickness of the soil layer
h_T	Thickness of top impeded boundary
h_B	Thickness of bottom impeded boundary
i	Hydraulic conductivity
i_1	Model parameter for non-Darcian flow equation
J_a	Mass rate of air
k	Vertical permeability of soil layer
k_j	Vertical permeability of the j^{th} layer
k_1	Vertical permeability of the top layer in two layerd soil strata
k_2	Vertical permeability of the bottom layer in two layerd soil strata
k_u	Ultimate permeability
k_T	Permeability of top-impeded boundary
k_B	Permeability of bottom-impeded boundary
k_{as}	Permeability of air phase in soil layer
k_{ws}	Permeability of water phase in soil layer
M	Slope of e-log k graph
m_v	Coefficient of volume compressibility
m_{v1}	Coefficient of volume compressibility of the top layer in two layerd soil strata
m_{v2}	Coefficient of volume compressibility of the top layer in two layerd soil strata
m_{1k}^s	Coefficients of overall volume change with respect to $(\sigma - u_a)$
m_{1k}^w	Coefficients of water phase volume change with respect to $(\sigma - u_a)$
m_{1k}^a	Coefficients of air phase volume change with respect to $(\sigma - u_a)$
m_2^s	Coefficients of overall volume change with respect to $(u_a - u_w)$
m_2^w	Coefficients of water phase volume change with respect to $(u_a - u_w)$
m_2^a	Coefficients of air phase volume change with respect to $(u_a - u_w)$
n	Porosity

q	Constant external load
q_0	Ultimate load in time-dependent loading
R_1	Normalized area of undissipated excess PWP attained at any arbitrary time during consolidation for non-uniform u_0 distribution.
R_0	Normalized area of undissipated excess PWP attained at any arbitrary time during consolidation for uniform u_0 distribution.
R_T	Model parameter of top impeded boundary
R_B	Model parameter of bottom impeded boundary
R_{Ta}	Model parameter of top impeded boundary air phase
R_{Ba}	Model parameter of bottom impeded boundary air phase
R_{Tw}	Model parameter of top impeded boundary water phase
R_{Bw}	Model parameter of bottom impeded boundary air phase
s^*	Maximum settlement at load q_u
s	Settlement of soil at the top surface
S	Degree of saturation
T	Temperature
t	Arbitrary time
t_o	Time (1 year)
t_c	Construction time
T_v,	Time factor
u_0	Initial excess pore water pressure
u_{0max}	Maximum value of initial excess pore water pressure
u	Excess pore water pressure
u_{max}	Peak position of the isochrones
U_{avg}	Average degree of consolidation
U_z	Degree of consolidation at any definitive depth
u_w	Pore pressure of water phase in unsaturated soil
u_a	Pore pressure of air phase in unsaturated soil
u_{a0}	Initial excess pore pressure of air phase in unsaturated soil
u_{w0}	Initial excess pore pressure of water phase in unsaturated soil
U_{avga}	The average degree of consolidation for the air phase
U_{avgw}	The average degree of consolidation for the water phase
V_a	The volume of free air dissolved air

V_d	Volume of free dissolved air
V_w	Volume of water
V_0	The total volume of soil
V_v	Volume of void
v	Velocity of water within the soil
z	Depth from the top surface
Z	Normalised depth with respect to the maximum flow path
z_1	The value of Z at the bottom layer

LIST OF ABBREVIATIONS

Symbol	Definition
CN method	Crank-Nicolson method
CDK	Constant loading, Darcian flow law and constant permeability
CDKV	Constant loading, Darcian flow law and variable permeability
CNDK	Constant loading, Non-Darcian flow law and constant permeability
CNDKV	Constant loading, Non-Darcian flow law and variable permeability
DF	Darcian flow
NDF	Non-Darcian flow
PWP	Pore water pressure
PTPB	Pervious top pervious bottom
PTPB	Pervious top impervious bottom
RDR	Rate of dissipation ratio
RDK	Ramp loading, Darcian flow law and constant permeability
RDKV	Ramp loading, Darcian flow law and variable permeability
RNDK	Ramp loading, Non-Darcian flow law and constant permeability
RNDKV	Ramp loading, Non-Darcian flow law and variable permeability
RL	Rectangular loading
TL	Trapezoidal loading
TRL	Triangular loading