Chapter 2
Literature review

Overview: The literature review has been systematically categorized into five key areas: TEG hot-side heat transfer enhancement, TEG device performance improvement, TEG cold-side heat dissipation strategies, optimization methodologies, and solar-assisted TEG waste heat recovery systems. The first section explores advanced heat transfer augmentation techniques, particularly the role of vortex generators in improving thermal energy absorption and enhancing TEG performance. The second section delves into critical design parameters that influence TEG efficiency, including novel thermoelectric materials, multi-stage configurations for better temperature gradient management, and optimized leg geometries to balance electrical and thermal transport properties. The third section focuses on shape-dependent performance analysis of nanofluids and hybrid nanofluids, emphasizing their effectiveness in improving thermal management and cooling efficiency on the cold side of the TEG. The fourth section discusses optimization approaches and predictive modeling techniques, particularly the integration of artificial neural networks for performance enhancement and parameter tuning in TEG systems. Finally, the last section examines solar-assisted TEG hybrid systems, highlighting their synergistic benefits and performance improvements in renewable energy applications. Through this structured review, a comprehensive understanding of the latest advancements and innovative strategies for optimizing TEG-based waste heat recovery systems is established.

The efficiency and performance of a TEG are primarily governed by the temperature difference between the hot and cold sides, as well as the TEG configuration, including the choice of materials, the number of stages, and leg geometry. To enhance heat transfer on the hot side, vortex generators have emerged as a simple yet effective solution for improving TEG performance. Additionally, advancements in TEG materials, multi-stage configurations, and various leg geometries have demonstrated significant improvements in power output. On the cooling side, the application of hybrid nanofluids has shown remarkable enhancement in thermal management, thereby boosting the electrical performance of TEGs. Consequently, the literature review for TEG performance enhancement has been structured accordingly, as illustrated in Fig. 2.1.

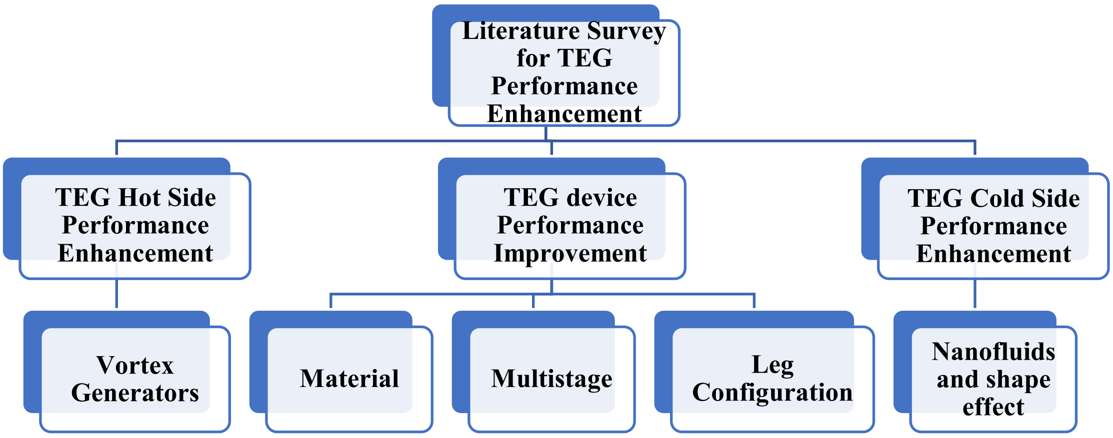


Fig. 2.1 TEG Performance Enhancement Literature Classification

2.1 TEG hot side performance enhancement

Enhancing heat transfer on the hot side of a thermoelectric generator (TEG) is crucial for maximizing its efficiency by maintaining a high-temperature gradient across the thermoelectric module. Effective heat transfer increases heat flux, reduces thermal resistance, and prevents excessive temperature drop, ensuring optimal energy conversion. Techniques such as extended surfaces, and turbulence promoters (vortex generators) improve convective heat transfer. These methods enhance fluid mixing, disrupt thermal boundary layers, and facilitate efficient heat delivery, ultimately leading to higher power output, improved system longevity, and a more compact, lightweight design. Heat transfer enhancement relies on key physical phenomena such as boundary layer disruption, increased turbulence, vortex formation, enhanced mixing, and increased surface area for heat exchange. Vortex generators (VGs) achieve this by introducing streamwise or transverse vortices into the flow, which promotes better mixing of hot and cold fluid layers, reducing thermal resistance and enhancing convective heat transfer. These vortices disrupt the thermal boundary layer, increasing fluid velocity gradients and improving heat transfer rates. Additionally, VGs induce localized flow acceleration and recirculation, leading to higher convective heat flux. By optimizing VG shape, size, and arrangement, heat transfer performance can be significantly improved while maintaining manageable pressure drop levels. Table 2.1 comprehensively illustrates the impact of vortex generator installation on heat transfer enhancement and fluid flow dynamics within heat exchangers.

Table 2.1: Vortex generator configurations for heat transfer enhancement

Authors	Vortex generators	Remarks
Caliskan (2014)	Punched triangular and punched rectangular VG in a rectangular channel	Maximum increment in average Nusselt number for punched triangular and punched rectangular were 55% and 51.3% respectively in comparison to the smooth channel for an attack angle of 45° (Reynolds number variation 3288 to 37,817).

Ebrahimi et al. (2015)	Rectangular VG in a rectangular channel microchannel	Nusselt number and friction factor increased by 2-25% and 4-30% respectively when Reynolds number was varied from 100 to 1100.
Ebrahimi et al. (2015)	Rectangular VG in microchannel	The enlargement of the recirculation zone behind the VGs, enhanced fluid mixing, and decreased thermal boundary layer thickness resulted in enhanced heat transfer performance at a high Reynolds number. Reynolds number was varied from 100 to 1100.
Wang et al. (2016)	Rectangular fins, cylindrical grooves, and deflectors in exhaust heat exchanger	The cylindrical grooves improved the heat transfer and reduced the back pressure. Furthermore, a portion of the heat exchanger uncovered with thermoelectric modules at the downstream section could make the upstream section hotter and make full use of each TEG.
Ahmed et al. (2017)	Triangular VG in a triangular duct	The maximum pressure drop increased was less than 10% while

		the heat transfer improved by 45.7% for 3 vol.% of Al_2O_3 at Re=16000.
Samadifar et al. (2018)	Simple rectangular, rectangular trapezoidal, angular rectangular, wishbone, intended, and waved VGs in a plate-fin heat exchanger	A simple rectangular vortex generator exhibited the highest gain in heat transfer performance of 7% and an angle of installation of 45° was found to be the most effective (Re=200).
Jiansheng et al. (2019)	Miniature cuboid VG in a rectangular channel	The synthesis performance of heat transfer and flow was enhanced by 8.15% relative to the smooth channel. Reynolds no. used was 3745.
Zhang et al. (2020)	Rectangular and V-shaped VG in a circular tube	The tube equipped with P-RWVGs exhibits an enhancement in heat transfer rate ranging from 54% to 118% and an increase in flow resistance between 152% and 568% compared to a smooth tube. Similarly, for the tube with V-RWVGs (V-shaped rectangular winglet vortex generator), the heat transfer rate improves by 60% to 118%, while the flow resistance

		rises by 141% to 644%. The experiments were conducted in the Reynolds number range of approximately 6000–20000.
Tian et al. (2020)	Triangular VG in a circular tube	For a thermally efficient heat exchanger, the location of the VGs concerning the center line walls plays an important role. The Reynolds number was considered to be between 4000 and 12000.
Carpio et al. (2021)	Delta VG in a compact heat exchanger	The minimum and maximum increase in Nusselt number due to vortex generators were observed to be 19% and 102% at Reynolds numbers 400 and 2000, respectively.
Demirag et al. (2022)	Conic VG in a rectangular channel	The Nusselt number improved by 58.75% in comparison to the smooth channel and an 8.42% increment in the thermal enhancement factor was observed for the lowest Reynolds number value (Re = 5000–20,000).
Das et al. (2023)	Butterfly-wing VG in a rectangular microchannel	The Nusselt number increased by 92%, 26%, 12%, 2.9%, and 2%

		when Reynolds number (142-544), upper width (0.24-0.36mm), wing height (0.2-0.35mm) lower width (0.12-0.24mm) and length of the vortex generators (1-3mm) were varied.
Wu et al. (2024)	V-winglet in a circular tube	Nusselt number increased in the range of 130.56-156.43% when the pitch (25-75mm) of the vortex generators was varied. The highest performance enhancement factor of 2.83 was obtained at a pitch of 25mm with 8 winglets (Re=7000-17000).

Thus, from the above literature, we observed how vortex generator configurations (shapes of VGs, distance between consecutive VGs, angle of inclination) help in the heat transfer enhancement in different applications. Karana and Sahoo (2021), investigated an exhaust heat exchanger experimentally to improve the output of a TEG-based WHR system by dividing the heat exchanger into computational domains (Niu et al., 2014) and employing the 1-D thermal resistance method in conjunction with relevant heat transfer correlations (Bergman et al., 2011). The uncertainties associated with the experiment were effectively calculated for all the parameters (Kline & Mcclintock, 1953). The maximum thermohydraulic efficiency factor achieved was 1.93. Furthermore, the study predicted a reduction in the twisted tape-equipped heat exchanger size for the same TEG power output as the smooth heat exchanger. Furthermore, Ma et al. (2015) assessed the feasibility of employing longitudinal vortex generators (LVGs) to enhance heat transfer in thermoelectric generators (TEGs). A coupled fluid-thermal-electric model was developed using COMSOL Multiphysics to analyze the influence of LVG height, LVG attack angle, and hot-side inlet gas temperature on TEG performance. The results indicated that the optimal performance was achieved when the LVGs extended across the full height of the channel at the highest examined temperature (550K). Under these conditions, heat input, net power output, and thermal conversion efficiency improved by 29%–38%, 90%–104%, and 31%–36%, respectively, compared to a smooth flow channel. Also, in a study, the influence of longitudinal vortex generators (LVGs) on the performance of a thermoelectric power generator (TEG) integrated with a plate-fin heat exchanger was investigated (Ma et al., 2017). The results revealed that the presence of LVGs induced complex three-dimensional vortices in the cross-section downstream of the LVGs, thereby enhancing heat transfer and electrical performance compared to a TEG without LVGs. Under baseline operating conditions, the heat input and open-circuit voltage of the TEG with LVGs were increased by 41–75% in comparison to a TEG with a smooth channel. In a study (Pal et al., 2012), simulations were performed on a heat exchanger with winglet vortex generators. They revealed a major heat transfer increase because the winglets created strong swirling vortices. This mixing effect thinned the thermal layer, enabling potential miniaturization of future heat exchangers. However, the aforementioned studies have not investigated the effect of vortex generator shape, distance-to-height ratio, and inclination angle on TEG performance.

2.2 TEG device performance improvement

The performance of a thermoelectric generator (TEG) is significantly influenced by the selection of p-type and n-type materials, the number of stages, and the geometry of the TEG legs. The choice of thermoelectric materials determines the Seebeck coefficient, electrical conductivity, and thermal conductivity, all of which directly impact the Figure of merit (ZT) and overall efficiency. High-performance materials with optimized transport properties enhance power output and conversion efficiency. Additionally, the number of stages in a TEG module affects its ability to handle large temperature gradients, with multi-stage configurations enabling higher thermal-to-electric conversion efficiency by optimizing temperature distribution across each stage. Furthermore, TEG leg geometry, including length, cross-sectional area, and aspect ratio, influences heat conduction, electrical resistance, and mechanical stability. Optimizing leg dimensions ensures minimal parasitic losses and maximized energy conversion. By strategically selecting high ZT materials, employing multi-stage architectures, and refining leg geometry, TEG performance can be significantly improved, leading to enhanced power generation and efficiency. Several literatures are available that give a deep insight into the above-mentioned practices to improve the TEG performance for waste heat recovery.

2.2.1 TEG materials

The development of new thermoelectric materials is crucial for enhancing the efficiency, scalability, and practical applicability of thermoelectric generators (TEGs). The performance of a TEG is primarily determined by the dimensionless Figure of merit (ZT), which depends on the Seebeck coefficient, electrical conductivity, and thermal conductivity of the material. Conventional thermoelectric materials, such as bismuth telluride (Bi_2Te_3), have reached performance limits in many applications, necessitating the exploration of alternative materials with superior thermoelectric properties. New materials can offer improved efficiency, reduced material costs, and better thermal and mechanical stability under varying operating conditions. Additionally, the search for environmentally friendly, abundant, and non-toxic materials is essential for large-scale deployment in waste heat recovery, energy harvesting, and industrial applications. Organic thermoelectric materials are gaining attention over conventional thermoelectric materials due to their low thermal conductivity, flexibility, lightweight nature, and cost-effective fabrication (Massetti et al., 2021). They are composed of earth-abundant, non-toxic elements, making them more sustainable and scalable. Additionally, their electronic properties can be tuned through molecular engineering, enabling tailored performance for specific applications. While challenges such as lower electrical conductivity and thermal stability remain significant hurdles for wider adoption, concerted advancements in nanomaterial design, composite engineering, and the development of sophisticated hybrid systems are steadily enhancing their viability for next-generation flexible electronics and energy harvesting applications. These advancements are enhancing their performance and durability, making them increasingly viable for next-generation applications in wearable technology and efficient energy harvesting from ambient sources. Achieving a ZT value greater than 1 is challenging because the material's electrical conductivity, thermal conductivity, and Seebeck coefficient are interdependent. Optimizing one often negatively impacts another. Low ZT (<1) results in poor energy conversion efficiency, generating insufficient electrical power or providing weak cooling, which severely limits practical applications for waste heat recovery. Substantial progress in thermoelectrics has been driven by a worldwide research effort focused on synthesizing high-performance materials, with notable breakthroughs in thin-film architectures (Venkatasubramanian et al., 2001), quantum-confined structures (Harman et al., 2002), and advanced semiconductors (Majumdar, 2004). Table 2.2 discusses some of the different thermoelectric materials being developed in recent times.

Table 2.2: Summary of different TEG materials

Author	TEG Material	Remarks
Sun et al. (2012)	poly[Kx(Ni-ett)] (Full name-poly(Nickel-1,1,2,2-ethylene tetrathiolate))	The n-type material exhibited a power factor of 147μW/mK^2 and a ZT value of 0.31 at 440K.
Kim et al. (2013)	PEDOT:PSS treated with Ethylene glycol (EG)/ Dimethyl sulfoxide (DMSO)	The Seebeck coefficient, electrical conductivity, and thermal conductivity obtained for the p-type material at room temperature were 73μV/K, 880S/cm, and 0.33W/mK respectively.
Zhao et al. (2014)	BiCuSeO oxyselenides	The Seebeck coefficient and electrical conductivity of the p-type material were 349μV/K and 1.12S/cm respectively.
Menon et al. (2015)	poly[Kx(Ni-ett)] (Full name-poly(Nickel-1,1,2,2-ethylene tetrathiolate))	A high-power density of 1–3mW/cm^2 was obtained after the material synthesis.
Sun et al. (2016)	Electrochemically treated poly[Kx(Ni-ett)]	The Seebeck coefficient, electrical conductivity, and thermal conductivity obtained for the n-type material were -150μV/K, 310S/cm, and 0.84W/mK respectively at 400K.

Chen et al. (2017)	Ni/PVDF (Ni nanowires within poly(vinylidene fluoride))	The Ni/PVDF nano-composites demonstrated unusual decoupling of electrical conductivity and the Seebeck coefficient concerning Ni content. With 80 wt% Ni at 380K, they achieve a peak power factor of 220μW/mK and a ZT value of 0.15.
Wang et al. (2018)	PEDOT/Bi_2Te_3	The p-type material exhibited a power factor of 1350μW/mK^2 and a ZT value of 0.58 at room temperature.
Karana and Sahoo (2019)	$Ag_{0.8}Pb_9Sn_9Sb_{0.6}Te_{20}$ and $Ag_{0.8}Pb_{19+x}SnSbTe_{20}$	Doping has a strong impact on TEG performance.
Chen et al. (2020)	$Bi_{0.4}Sb_{1.6}Te_3$	Optimized TEG gives 21.95% higher power than equally segmented TEG.
Chen et al. (2021)	$Ag_2Se_{1-x}Te_x$ (x = 0.1, 0.2, 0.3, 0.4, and 0.5)	Ultralow lattice thermal conductivity in the range of 0.21−0.31W/mK at 300K.
Jiang et al. (2022)	PEDOT:PSS	The highest power factor of 86.3μW/mK^2 measured at 313K was obtained. It also exhibits excellent environmental stability

		with less than 10% variation in resistance for 28 days at room temperature.
Liu et al. (2023)	PEDOT:PSS flexible thin films	The power factor of PEDOT:PSS films was improved by using a water-based solution. A high power factor of 64.4μW/mK^2 is achieved in PEDOT:PSS films at 360K.
Xia et al. (2024)	PEDOT:PSS/SWCNTs	Ultrahigh power factors of >500 and 185μW/mK^2 in p- and n-type hybrid films respectively.

2.2.2 TEG multi-staging

The number of stages in a thermoelectric generator (TEG) plays a crucial role in managing large temperature differentials effectively. In a multi-stage configuration, thermoelectric elements are arranged in a series of cascaded layers, where each stage operates within an optimized temperature range. This staged approach reduces thermal losses, enhances energy conversion efficiency, and prevents excessive thermal stress on individual thermoelectric materials. By strategically distributing the temperature gradient across multiple stages, multi-stage TEGs can achieve improved thermal-to-electric conversion efficiency compared to single-stage systems, making them particularly beneficial for applications involving high-temperature heat sources. Table 2.3 comprises multiple pieces of literature showcasing the benefits of increasing stages in a TEG module.

Table 2.3: TEG multi-staging

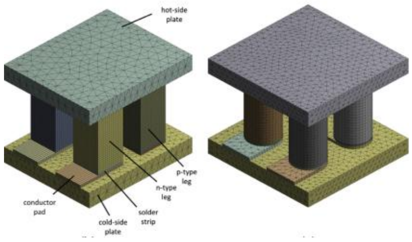
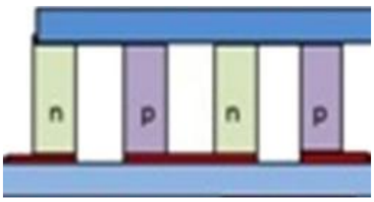
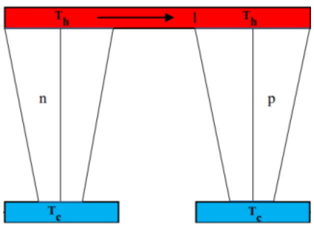
Author	TEG leg geometry	Remarks
Arora et al. (2015)	Single and two-stage	Multi-objective optimization depicts improvement in TEG performance by multi-staging.
Arora et al. (2016)	Two-stage	The efficiency performance of TEG improved by 16.6% for two-stage configurations.
Ahmadi Atouei et al. (2017)	Two-stages	The proposed TEG system gives 27% higher electrical potential than a single-stage system.
Ahmadi Atouei et al. (2017)	Multi-stage	Maximum power density rises by 79.08%, and maximum conversion efficiency increases by 96.47% when TEG is multi-staged.
Cheng et al. (2018)	Multi-stage	The maximum percentage increment in conversion efficiency and power densities were 96.47% and 79.07% respectively.
Sun et al. (2019)	Two-stages	Two-stage TEG system results in power and efficiency values of 1.96W and 10.14% respectively.

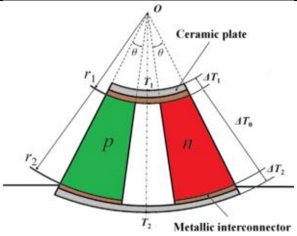
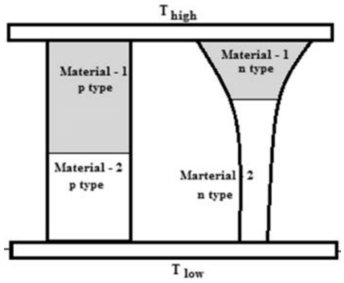
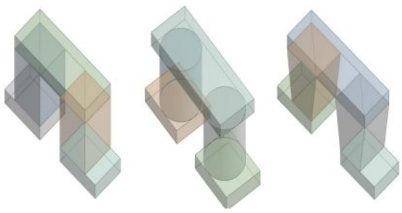
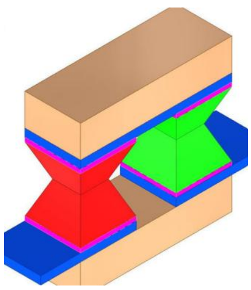
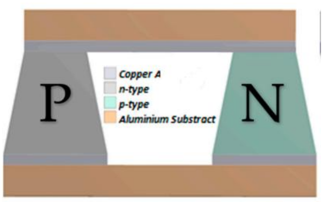
Zhao et al. (2020)	Two-stages	Maximum power density and efficiency upsurges by 4.33% and 64.25% respectively with two-stage TEG.
Yin et al. (2021)	Multi-stage	Compared to the single-stage TEG, a remarkable increase of 36.40% for output power and 34.47% for conversion efficiency is achieved respectively in the multi-stage.
Maduabuchi et al. (2022)	Single and multi-stage	The proposed TEG with tapered legs (trapezoidal and X-legs) improves the exergetic efficiency.
Qi et al. (2023)	Two-stage	Two stages give the maximum power and efficiency of 9.872W and 4.9% respectively.
Yang et al. (2024)	Two-stage	Maximum single and two-stage TEG conversion efficiencies are 1.9% and 2.6% respectively.

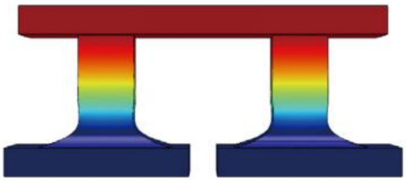
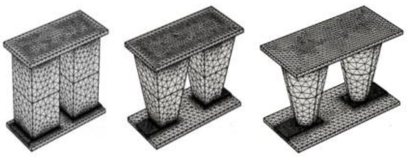
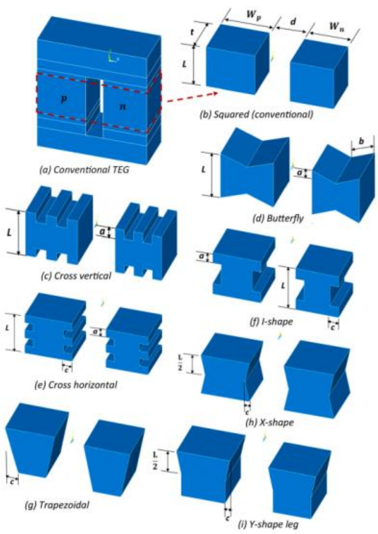
2.2.3 TEG leg geometry

The geometry of TEG legs, including their length, and cross-sectional area plays a critical role in determining the device's thermal and electrical performance. The leg dimensions directly influence heat conduction, electrical resistance, and mechanical integrity. Longer legs increase thermal resistance, reducing heat leakage but also raising electrical resistance, which can lower power output. Conversely, a larger cross-sectional area decreases electrical resistance, improving electrical conductivity but also increasing heat conduction, which may reduce the temperature gradient necessary for efficient energy conversion. Therefore, careful optimization of leg geometry is essential to minimize losses, maintain structural stability, and enhance overall energy conversion efficiency. Different literature exhibiting the impact of TEG leg geometry has been compiled in Table 2.4.

Table 2.4: TEG leg geometries

Author	Stages	Remarks
Erturun et al. (2015)	Rectangular and cylindrical 	Power output and conversion efficiency increase with increasing leg width and decreasing leg length.
Arora et al. (2016)	Rectangular 	The optimized series and parallel configurations gave output powers of 3.42W and 3.85W respectively.
Lamba et al. (2017)	Flat and trapezoidal 	Trapezoidal geometry improves the energy and exergy efficiency of the device.
Zhang et al. (2018)	Annular leg	The maximum output power per unit mass was attained only

		when the cross-section area of the thermoelectric leg was constant for the ideal annular TEG.
Karana and Sahoo (2019)	Asymmetrical and segmented leg 	The output current and overall efficiency of the segmented TEG increases approximately by 33.3% and 5% when the temperature ratio is varied from 0.45 to 0.55.
Lee et al. (2020)	Square, cylindrical, and trapezoidal 	Trapezoidal geometry emerged as the best leg geometry with output power and conversion efficiency of 0.73W and 13.2% respectively.
Maduabuchi et al. (2021)	X-leg 	It was highly recommended that X-legs should be adopted whether in single or double-stage Solar-TEG configurations.
Ramos-Castañeda et al. (2021)	Rectangular and trapezoidal 	The maximum efficiency between the five geometry types was 5.53%, with a substrate of 110 × 100mm^2.

Ge et al. (2022)	Internal arc 	The internal arc X exhibited a maximum conversion efficiency of 3.28%.
Khalil et al. (2023)	Rectangular, pin, and cone 	Rectangular leg produced the highest power and conversion efficiency of 0.3019W and 12.47% for leg height of 1 and 4.5mm respectively.
Aljaghtham (2024)	Nine geometries (Rectangular, cylindrical, etc.) 	The cross-vertical and butterfly-shaped leg exhibited 31% higher energy efficiency than the conventional leg.

Thus, the above literature comprehensively explores the role of TEG material, leg geometry, and stages. It becomes important to take such measures to improve TEG performance and make it compatible with wider temperature applications. Sun et al. (2014), studied two-stage serial and parallel thermoelectric generator models using temperature-dependent bismuth telluride and skutterudite materials with an internal combustion engine's exhaust as the heat source. Results show heat source temperature is crucial for design choice, with two-stage systems outperforming single-stage ones in power, efficiency, and exergy efficiency at higher temperatures. Sahoo and Karana (2020), analyzed a novel tapering segmented pin design for thermoelectric generators using modified bismuth and lead telluride materials, highlighting that higher load resistance ratios improve efficiency, with exergy efficiency strongly influenced by the shape factor. Furthermore, carbon-based semiconductors have garnered significant interest as promising thermoelectric materials for low-temperature energy harvesting due to the plentiful availability of their constituent elements, simplicity in fabrication processes, and naturally low thermal conductivity. For instance, PEDOT:PSS/SWCNTs films were explored by Xia et al. (2024), and obtained very high power factor of over 500 and 185μW/mK^2 and normalized power density of 2.5μW/cm^{2}K^2. Rathi et al. (2024), observed a notable improvement in the thermoelectric performance of PEDOT by developing an innovative ternary composite film incorporating Bi_2Te_3. The composite films exhibited increased Seebeck coefficient, electrical conductivity, and power factor compared to pure PEDOT:PSS films, with Bi_2Te_3 contributing higher Seebeck coefficients due to its intrinsic thermoelectric characteristics. Furthermore, pieces of literature (Liu et al., 2017, Sun et al., 2016) suggested that poly[Kx(Ni-ett)] is a novel thermoelectric material as it is lightweight, mechanically flexible, and derived from abundant resources, can be fabricated using cost-effective solution-based methods, making them more suitable than their inorganic counterparts for powering portable and wearable electronics. Because of the above-mentioned advantages based on properties and applications, the novel organic thermoelectric materials (PEDOT:PSS, PEDOT:PSS/ Bi_2Te_3, and poly[Kx(Ni-ett)]) with trapezoidal leg geometry for two stages have been found reasonable for TEG applications.

2.3 TEG Cold side heat transfer enhancement

2.3.1 Shape-based performance of nanofluids and hybrid nanofluids

Efficient cooling on the cold side of a TEG is essential to maintain a high-temperature difference (ΔT), which directly influences power generation efficiency and system longevity. Hybrid nanofluids, composed of multiple nanoparticles dispersed in a base fluid, enhance cooling by improving thermal conductivity and convective heat transfer, ensuring better heat dissipation. The shape of nanoparticles plays a crucial role in heat transfer and fluid flow, with elongated and high-aspect-ratio particles (e.g., cylindrical, platelets, cubic) offering superior thermal transport but potentially increasing viscosity and flow resistance. Optimizing nanoparticle shape and composition in hybrid nanofluids enables enhanced cooling performance, leading to improved TEG efficiency and energy conversion. Table 2.5 covers the studies dedicated to ascertaining the impact of nanoparticle shape on the coolant performance.

Table 2.5: Nanoparticles and their shape effects

Authors	Nanoparticles	Remarks
Elias et al. (2014)	Boehmite alumina (γ-AlOOH) Shapes: cylindrical, bricks, blades, and platelets	Maximum overall heat transfer coefficient is observed for cylindrical shaped nanoparticles at 1% vol. fraction.
Ellahi et al. (2015)	Cu Shapes: cylindrical, bricks, and platelets	Nusselt number increased by 17.53%, 11.55%, and 21.39% for cylindrical, brick, and platelet shape-based nanofluid for φ=0.08 than the base fluid.

Reddy et al. (2016)	Al_2O_3, TiO_2 Shape: Spherical	The temperature profile enriches due to an increase in the thermal boundary layer with the introduction of nanoparticles.
Khan et al. (2017)	Cu Shapes: platelet, cylindrical, and brick	The platelet-shape-based nanofluid performs better thermally followed by cylindrical and brick-shaped nanoparticles.
Liu et al. (2018)	Al_2O_3 Shapes: Sphere, platelet, blade cylinder, brick	For the nanofluid containing platelet-shaped particles with a 2% volume fraction, the Nusselt number increases up to 38.9% compared with base fluid followed by nanofluids containing nanoparticles with cylinder, blade, sphere, and brick shapes.
Kumar et al. (2019)	Al_2O_3, CuO Shapes: spherical, cylindrical, brick, platelets, and blades	The thermal conductivity is observed to be higher in spherical and cylindrical nanoparticle shapes and then followed by bricks, blades, and platelet shape nanoparticles. The dynamic viscosity of platelets shapes hybrid nanofluid is found to be maximum and followed by

		cylindrical, blades, bricks, and spherical shapes.
Benkhedda et al. (2020)	TiO_2, Ag Shapes: spherical, cylindrical, platelets and blades	Hybrid nanofluid with a blade-blade shape combination exhibited the highest Nusselt number of 42.3 at φ=0.06 and Re=1275.
Kumar et al. (2021)	Al_2O_3, CNT, graphene Shapes: spherical, cylindrical, and platelet	Hybrid nanofluid with spherical and platelet shape particles has 2.94% higher effectiveness compared to spherical and cylindrical shape nanoparticle-based hybrid nanofluid.
Ramzan et al. (2022)	Graphene (cylindrical), Ag (platelet), and CuO (spherical)	The heat dissipation capacity of graphene-Ag/H_2O is higher compared to that of graphene-CuO/H_2O. Additionally, in solar thermal energy systems, the Graphene-Ag/H_2O hybrid nanofluid containing cylindrical and platelet-shaped particles demonstrates superior performance compared to a combination of cylindrical and spherical particles.
Chu et al. (2023)	Au, Ag Shapes: blade, platelet, cylindrical, and brick	The velocity rate for brick-shaped nanoparticles is higher than the

		cylinder, platelet, and blade-shaped nanoparticles.
Yahyaee (2024)	Al_2O_3 Shapes: spheres, bricks, blades, cylinders, and platelets	The highest increment in thermal conductivity is for blade-shaped nanofluid (1.38%) while the maximum increment in kinematic viscosity was observed in platelet-shaped nanofluid (10.85%).

2.3.2 Mini channel heat sinks for nanofluids and hybrid nanofluids

Numerous researchers have explored diverse heat sink configurations to enhance the cooling efficiency of various heat-dissipating devices (Tuckerman and Pease, 1981). Advanced studies have focused on optimizing heat sink design (Shamsuddin et al., 2021), conducting comprehensive thermal (Yang et al., 2017) and hydraulic modeling (Bejan, 2013), performing energy and exergy analyses (Mahmoud et al., 2021), and evaluating key performance parameters (Liu et al., 2020), all while integrating practical considerations to ensure real-world applicability. For instance, Kumar and Sarkar (2019), investigated the heat transfer and pressure drop characteristics of a mini channel heat sink, experimentally and numerically, using hybrid nanofluids (volume concentration-0.1%). The heat sink comprised nine parallel rectangular mini channels, each with a depth of 3mm and a width of 1mm. The study observed a maximum enhancement in the convective heat transfer coefficient of 8.5% in numerical simulations and 12.8% in experimental tests when utilizing the Al_2O_3 (10:0) hybrid nanofluid. Bahiraei et al. (2019), evaluated the thermohydraulic performance of a hybrid nanofluid containing graphene–silver nanoparticles in a microchannel heat sink with ribs and secondary channels. The findings revealed that integrating nanofluids, ribs, and secondary channels significantly enhanced heat sink efficiency.

Moreover, the convective heat transfer coefficient increased with higher nanoparticle concentration and Reynolds number, achieving a 17% improvement when the concentration rose from 0 to 0.1% at Re=100. In another study, the thermal efficiency of an aluminum mini channel heat sink with a rectangular cross-section was examined experimentally (Ataei et al., 2020). Distilled water, TiO_2–water nanofluid, Al_2O_3–water nanofluid, and a hybrid Al_2O_3/TiO_2–water nanofluid, each with a volume concentration of 0.5%, served as cooling fluids. A constant heat flux boundary condition was applied and sustained by a 36W heater positioned at the base of the heat sink. The convective heat transfer coefficient improved by up to 16.97% in comparison to pure water. Additionally, the wall temperature dropped by as much as 5°C when employing the Al_2O_3/TiO_2–water hybrid nanofluid instead of pure water. Also, the impact of article mixture ratios in hybrid nanofluids on mini channel heat sink performance was experimentally examined (Kumar and Sarkar, 2020). Water-based hybrid nanofluids were formulated with a total volume concentration of 0.01% using Al_2O_3 and MWCNT nanoparticles in different proportions (10:0, 8:2, 6:4, 4:6, 2:8, and 0:10). The highest improvements in heat transfer coefficient and pressure drop, recorded at 44.1% and 68.1%, respectively, were observed for the MWCNT nanofluid.

Hybrid nanofluids enhance TEG performance by significantly improving heat transfer at the hot side. Their superior thermal conductivity extracts more waste heat, creating a steeper temperature gradient across the TEG modules. This directly increases the voltage and power output according to the Seebeck effect. For instance, Selimefendigil et al. (2021) simulated a TEG cooled by nanofluids. The Ag/MgO-water hybrid nanofluid achieved the highest power output, up to 9.3% at Re=500. Performance improved with Reynolds number and nanoparticle concentration. CNT-water was more effective at higher flow rates (Re=1500), yielding a 6.6% power increase. A review research (Garud et al., 2021) summarized using nanofluid-cooled TEGs for PV thermal management. The TEG harvests PV waste heat for extra power, while nanofluids enhance cooling. This integrated PV/T-TEG system synergistically improves overall electrical efficiency and performance, serving as a reference for future renewable energy innovations. Lastly, a study (Khatirzad & Sheikholeslami, 2025) found that combining pin/plate fins with MWCNT/SiC-water nanofluid cooling optimizes TEG performance. This configuration increased power output by 6.9% over standard plate fins and by 2.6% over water cooling alone, highlighting the critical role of advanced thermal management in enhancing energy harvesting. Although the use of hybrid nanofluids for improved TEG cooling has been studied, the performance enhancement attributable specifically to nanoparticle shape on the cold side represents a significant research gap.

Thus, for hot and cold sides of the TEG, vortex generators and hybrid nanofluids significantly enhance TEG performance by synergistically managing heat transfer on both sides of the module. On the hot side, vortex generators intensify convective heat transfer by disrupting thermal boundary layers and promoting turbulence, thereby increasing thermal energy input. Conversely, on the cold side, hybrid nanofluids, with their superior thermal conductivity, drastically improve waste heat extraction, which is critical for maintaining a high temperature gradient. Cited studies confirm their individual and combined efficacy in boosting heat transfer coefficients and, specifically, TEG power output. Furthermore, within the discussion on cold-side enhancement, additional research has been referenced to explicitly illustrate the established connection between the application of nanofluids and measurable performance gains in TEG systems.

2.4 Optimization methodology

Optimization techniques help in solving engineering problems by systematically finding the best possible solutions while considering constraints, efficiency, and performance metrics. These methods, such as genetic algorithms, gradient-based approaches, and machine learning-driven optimization, enhance design, resource utilization, and decision-making across various engineering applications. Various optimization techniques have been employed for studies related to vortex generator-based heat transfer enhancements in the past. Das and Hiremath (2023) numerically investigated a novel butterfly-wing vortex generator (VG) inside a microchannel. They utilized the Taguchi design of experiments to analyze thermohydraulic performance and entropy generation, employing an L_{27} orthogonal array with five factors and three levels. Signal-to-noise (S/N) ratio analysis and the TOPSIS decision-making method were applied to identify the optimal parameters from the Pareto front. The best input conditions were found to be w=0.48mm, b=0.12mm, l=1mm, h=0.38mm, and a Reynolds number of 544, yielding output responses of Pf =1.35 and $N_{s,a}$=0.67. Similarly, Feng et al. (2023) conducted computational studies on innovative VG designs, including longitudinal VGs, dimples/protrusions, and grooves, to enhance the airside performance of H-type finned tube heat exchangers. Using the Taguchi method, they examined the influence of geometric parameters on heat transfer, flow resistance, and overall thermal-hydraulic efficiency. At Reynolds numbers ranging from 4,650 to 28,300, the best design showed an improvement of 0.9–23.8%, 24.5–57.1%, and 8.3–37.5% over LVGs, dimples/protrusions, and grooves, respectively. A numerical study (Zeng et al., 2010) analyzed the effects of attack angle, VG length and height, fin material, fin thickness, fin pitch, and tube pitch on the performance of a vortex-generator fin-and-tube heat exchanger. The Taguchi method was used for optimization, revealing that six key factors significantly influenced the JF factor. Two optimal conditions (A1B3C3D2E1F2G1H3 and A2B2C2D3E1F2G1H3) were identified, and their reproducibility was confirmed through analytical results. Additionally, a performance enhancement of 4.5–26.6% was observed compared to reference cases. Several other studies (Zhang et al., 2019, Xie et al., 2022, Li et al., 2023, Liang et al., 2023) have also focused on optimizing vortex generators to improve heat transfer efficiency. Hence, the optimization of different VG shapes in heat exchangers under different ambient conditions should be explored.

2.5 Solar-assisted TEG hybrid WHR systems

TEGs have also been integrated with solar-driven devices. Gharzi et al. (2023) proposed a parabolic trough collector-TEG (PTC-TEG) hybrid system, using pressurized heat transfer fluid (HTF) and TEG modules, achieving a 15.75% efficiency enhancement, combining improved thermal performance and additional electrical generation from absorbed solar radiation. A bidirectional solar thermoelectric generator (STEG) coupled with a latent heat storage and cooling system (LHSCS), proposed by Montero et al Montero et al. (2023) enables 24-hour power generation by storing waste heat during the day and using it at night. Experimental results showed 5% efficiency, 5735Wh annual electricity generation, and competitive costs compared to PV systems, offering a sustainable solution. Alobaid et al. (2023) evaluated frustum leg thermoelectric generators (FLTEGs) against trapezoidal and rectangular-leg designs using validated ANSYS simulations and neural networks. FLTEGs showed 5.7% higher power, 5.6% greater efficiency, and 2.7% reduced thermal stress under 100 Suns. Neural networks predicted performance 702 times faster, minimizing stress to 0.67GPa in optimized designs. Thus, researchers have studied SWH-TEG coupled systems (He et al. 2012, Zhang et al., 2013, Manivannan et al., 2022) in various aspects over the years. An efficient solar water heater (Zhang et al., 2024) and TEG (Shen et al., 2016) hybrid system with proper validation (Khalil and Hassan, 2020) must be designed for further performance analysis. Although TEGs have been successfully integrated into solar water heating systems for improved efficiency and waste heat recovery, no study has yet developed a hybrid system equipped with an optimized vortex generator for the TEG's hot side.

2.6 Highlights

The literature survey has provided comprehensive insights into advanced methodologies for efficient waste heat recovery using thermoelectric generators (TEGs). Researchers have explored various strategies through both experimental and analytical approaches, thoroughly examining all critical aspects of TEG systems, including the hot side, the thermoelectric module itself, and the cold side. On the hot side, the incorporation of vortex generators has been shown to enhance convective heat transfer, thereby improving thermal energy input to the TEG. A detailed analysis of the TEG module has highlighted the significance of key design parameters, such as novel thermoelectric materials with improved Figure of merit (ZT), multi-stage configurations for optimized temperature gradients, and leg geometry modifications to balance electrical conductivity and thermal resistance. Additionally, on the cold side, the thermohydraulic performance of hybrid nanofluids has been investigated, revealing the influence of nanoparticle shape on heat transfer enhancement and fluid flow characteristics. These advancements collectively contribute to the development of a highly efficient, compact, and optimized TEG-based waste heat recovery system, paving the way for enhanced energy harvesting and sustainability in thermal management applications. Moreover, optimizing TEG parameters and integrating TEG with solar appliances pave the way for advanced research and technological advancements.

The core novelty of this work lies in the comprehensive and integrated investigation of multiple advanced strategies for TEG performance enhancement, which includes the study of novel vortex generators for hot-side heat transfer, a multi-stage TEG architecture, and a cold-side analysis using hybrid nanofluids with nanoparticle shape effects. While these components are studied individually, their novel integration within a single research framework, culminating in a hybrid solar-TEG system with temperature-dependent properties, provides unique and practical design guidance that extends beyond the current state of the art.

2.7 Research gaps and scope

Despite extensive research efforts, significant advancements are still required to make this technology commercially viable. The literature review clearly identifies key knowledge gaps, such as the lack of detailed studies on vortex generator configurations (including novel shapes, spacing, and angles) and their impact on TEG performance. It also highlights the absence of research combining advanced TEG materials, leg geometries, and multi-staging for synergistic improvements. Additionally, while hybrid nanofluids enhance cold-side cooling, their nanoparticle shape-dependent effects remain unexplored in TEG applications. The review further notes the lack of studies integrating Taguchi DOE for optimization with ANN-based predictive modeling, as well as the untapped potential of EUSWH-TEG systems with fishtail vortex generators. These gaps will guide future research. Identified research gaps highlight the need for a comprehensive analysis of TEG-based waste heat recovery (WHR) systems. This thesis aims to bridge these gaps by focusing on performance enhancement through advancements in TEG device architecture, thermal management on both hot and cold sides, and the development of a TEG-based hybrid system for simultaneous power generation and water heating. Key performance parameters, including power output, conversion efficiency, heat transfer coefficient, Nusselt number, and pressure drop, are systematically evaluated using numerical and experimental approaches. The critical research gaps identified are as follows:

- For the hot side of the TEG, few vortex generator shapes have been analyzed in the literature. However, a detailed comparative analysis of vortex generator configurations (novel shapes, distance-to-height ratios, and angles of inclination of the VGs) along with its impact on TEG performance has not been found.
- Many scopes utilizing the application of novel TEG materials, different TEG leg geometry, and TEG multi-staging exist individually. However, utilizing the synergetic effect of all the aforementioned beneficial TEG aspects has not been previously done.
- Hybrid nanofluids improve the heat transfer rate on the cold side of the TEG and the shape of the nanoparticles play an important role in dictating the thermophysical properties of the nanofluid. However, this characteristic has not been employed to improve TEG performance on the cold side of the TEG.
- Open literature is not available where finding optimized TEG configuration using robust Taguchi DOE and further developing a prediction model utilizing Artificial Neural Network (ANN) has been performed.
- An Evacuated U-tube Solar Water Heater (EUSWH)-based TEG waste heat recovery system utilizing novel fishtail VG has not been explored in the literature available.