Chapter 5

Role of thermal noise for the growth kinetics of active model B (off-critical mixture)

Abbreviations/Acronyms: **AMB**(active model B), **AMBN**(active model B with noise), **PMB**(passive model B).

5.1 Introduction

Previous chapter, 4, was focused on the study of role of noise on the ordering kinetics of the collection of active Brownian particles modeled using coarse-grained conserved active model B (AMB) for the critical composition i.e. the average order parameter $\psi_0(\mathbf{r},t) = 0$. Recent studies Pattanayak et al. (2021a,b) have shown that off-criticality has a significant effect on the domain growth in AMB. In the current chapter, we study the system for the off-critical composition, that implies $\psi_0(\mathbf{r},t) \neq 0$. We model it for two cases (i) When the system is slight away from the critical point. We call it as slightly off - critical composition. (ii) When the system is far away from the critical composition. We call it as deep off-critical composition. The model is same as discussed in previous chapter 4. Here we study the case of off-critical composition, for different noise strengths η_a. For the critical composition, $\psi_0 = 0$, system is symmetric about the $\lambda = -\lambda$. Hence all the results will remain same by changing the sign of λ. But when $\psi_0 \neq 0$, then the system breaks the symmetry of $\lambda = -\lambda$ and now the the system maintains the symmetry by $(\lambda, \psi_0) \equiv (-\lambda, -\psi_0)$. Thus if the product of $\lambda\psi_0 > 0$, then the symmetry is maintained else not. Hence we independently consider two cases, $\lambda\psi_0 > 0$ and $\lambda\psi_0 < 0$ or when the activity, λ and off-criticality ψ_0 have same sign and when they are opposite in sign. We find that the sign of the product of activity, λ and off-criticality ψ_0, is one of the governing factors for the domain growth in the system.

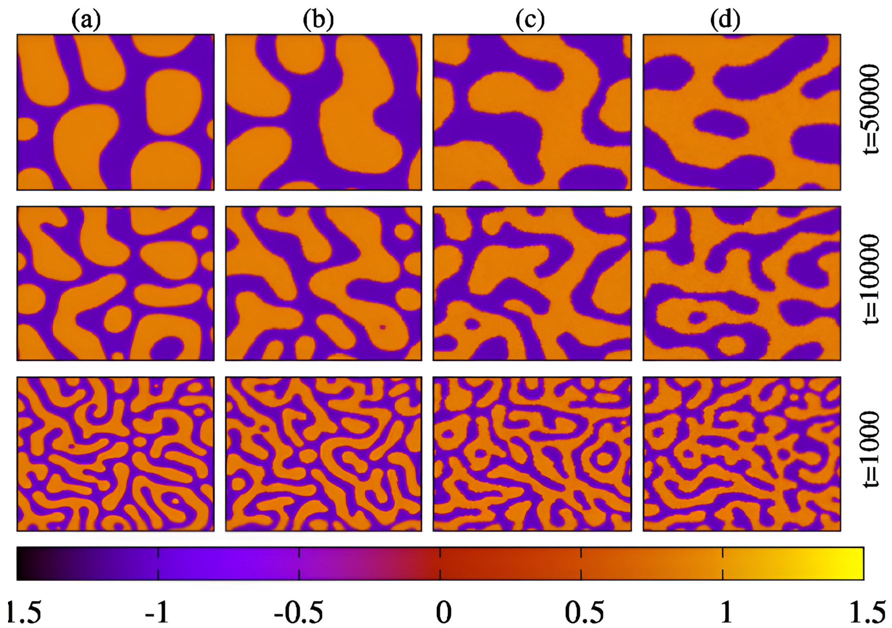

Fig. 5.1 (color online) Time evolution of the system for different values of noise strengths η_a, keeping the activity, λ fixed to 0.5. (a),(b),(c) and (d) are for noise strengths 0.2, 0.5, 0.8 and 1.0 respectively. The average order parameter value is $\psi_0 = 0.05$, that corresponds to slightly off-critical composition. The color bar shows the range of ψ value.

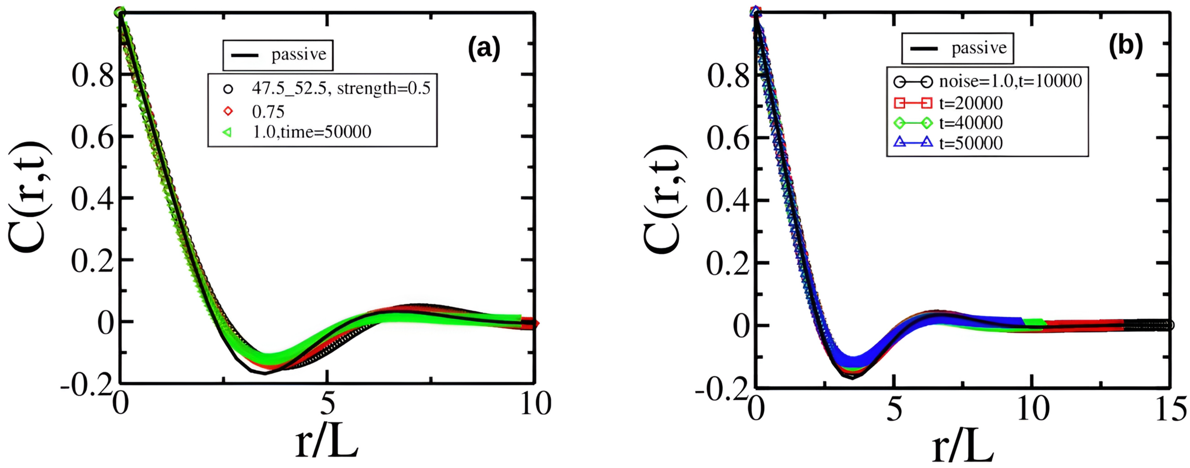

Fig. 5.2 (color online) shows the (a) static scaling for different strengths of noise $\eta_a = 0.5, 0.75, 0.1$ at fixed time $t = 50000$ for activity, $\lambda = 0.5$ and (b) dynamic scaling for noise strength 1.0 . Black solid line shows the correlation for PMB.

Here we start discussing our results for different cases. First we discuss the slightly off-critical mixture, $\psi_0(\mathbf{r},t) = \pm 0.05$ for the same and opposite sign of activity and off-criticality. Then we do the same for deep off-critical composition $\psi_0(\mathbf{r},t) = \pm 0.4$.

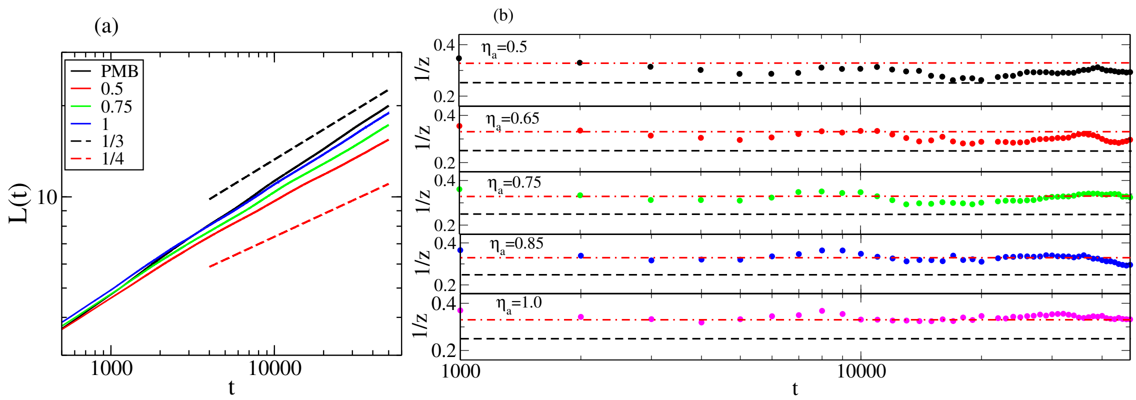

Fig. 5.3 (color online) (a) Domain length is shown for noise strengths $\eta_a = 0.5, 0.75$ and 1.0 is shown here. Black solid line shows the PMB. Black and red dashed lines are lines of slope $1/3$ and $1/4$ respectively. (b) shows the effective growth exponent for noise strengths $\eta_a = 0.5, 0.65, 0.75, 0.85$ and 1.0. Black and red dashed horizontal lines are to show the value 0.33 and 0.25 respectively

5.2 Results

5.2.1 Slightly off-critical composition (same sign, $\lambda\psi_0 > 0$)

Domain morphology and growth kinetics

First we observe the time evolution of the growing domains. Fig. 5.1 is the time evolution snapshots of the local density $\psi(\mathbf{r},t))$ of the particles. The particles rich regions and particles empty regions are represented by the color bar, blue (dark) and orange (bright) regions respectively. The three rows in Fig. 5.1 are for three different times $t = 1000$, 10000 and 45000 and different columns are for different strengths of noise. For all noise strengths $\eta_a = 0.2$, 0.5, 0.8 and 1, starting from the initial random homogeneous mixed state, the domains of ψ rich regions start to grow with time. We observe that for very small value of noise strength $\eta_a \leq 0.5$, the domains are elongated in nature. But for sufficiently high noise strengths, the domains become bi-continuous at late times (as shown in 5.1 top panel).

Next, we observe the growth kinetics. We first calculate the order parameter two-point correlation function as we have discussed in previous chapter 4. The order parameter two-point correlation function is defined as

$$C(r,t) =< \psi(\mathbf{r}_0+\mathbf{r},t)\psi(\mathbf{r}_0,t) > \tag{5.1}$$

where the angular brackets $< .. >$ denotes the average in all four directions, over reference position $\mathbf{r}_0$ and 100 independent realisations. Fig 5.2(a) and (b) show the scaled static and dynamic correlation functions respectively. The static correlation function is shown for different strengths of noise $\eta_a = 0.5, 0.75$ and 1.0 at fixed time $t = 50000$ and the dynamic scaling for fixed strength of noise $\eta_a = 1.0$. To compare with the result of PMB, we have also shown the plot for PMB at the same time, which is represented by

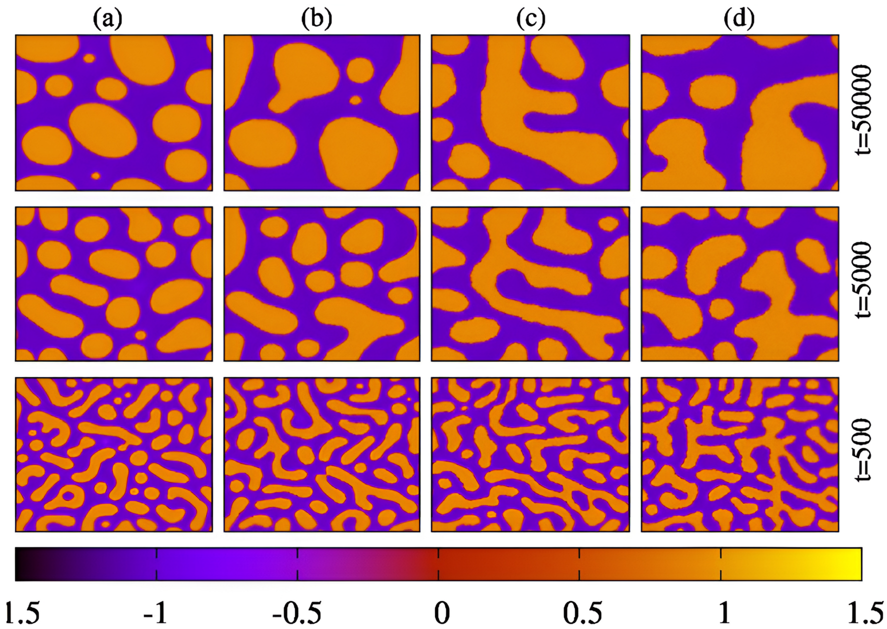

Fig. 5.4 (color online) Time evolution of the system for different values of noise strengths η_a, keeping the activity, λ fixed to 0.5. (a), (b), (c) and (d) are for noise strengths 0.2, 0.5, 0.8 and 1.0 respectively. The average order parameter value is $\psi_0 = -0.05$. The color bar shows the range of ψ value.

the black solid line. We find that the system does not show the static scaling and for higher noise strength, it approaches to scaled $C(r,t)$ of PMB. Also the system shows good dynamic scaling and scaled $C(r,t)$ is close to PMB.

Now to find out the growth exponent, we define characteristic length $L(t)$ as distance over which the correlation function, $C(r,t)$ falls to 0.5 of its maximum value at $r = 0$ as defined in chapter 4. We calculate the effective growth exponent, $\frac{1}{z_{eff}}$ as a function of time t, which is defined as

$$\frac{1}{z_{eff}} = \frac{d(lnL(t))}{d(lnt)} \tag{5.2}$$

We plot $L(t)$ vs. t in fig. 5.3(a) on log-log scale for $\eta_a = 0.5$, 0.75 and 1.0. We keep the value of activity λ fixed to 0.5. We also include the plot of PMB shown by the black solid line. The previous studies Puri (2009) Pattanayak et al. (2021a,b); R.Wittkowski & Cates (2014) have shown that the ordering kinetics of domain growth shows a growth law scales

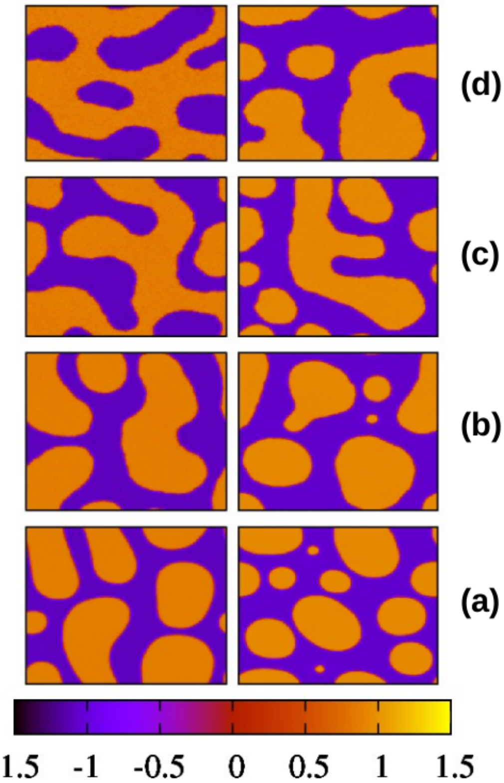

Fig. 5.5 (color online) Snapshots at late times are shown to compare the two cases for same and opposite sign of activity, λ and off-criticality ψ_0. We have taken $\lambda = 0.5$ and $\psi_0 = \pm 0.05$. Left column is for $\lambda\,\psi_0 > 0$ and right column is shown for $\lambda\,\psi < 0$ at time, $t = 50000$. (a),(b),(c) and (d) are for noise strengths 0.2, 0.5, 0.8 and 1.0 respectively. The color bar shows the range of ψ value.

as $L(t) \sim t^{1/z}$, where z is the dynamic growth exponent. We have shown in the previous chapter 4 that for critical mixture, the asymptotic growth exponent comes as $z = 3$ for all strengths of noise. In our present study, we find that as we increase the strengths of noise, the growth exponent approaches to $z = 3$. However, for intermediate time and small strength of noise $\eta_a \leq 0.65$, it shows a deviation from $z = 3$. In fig 5.3(b), we plot $\frac{1}{z_{eff}}$ vs. time, t for the noise strengths $\eta_a = 0.5, 0.65, 0.75, 0.85$ and 1.0. The two dashed black and red lines show the value 0.25 and 0.33 respectively. We find that for small strengths of noise $\eta_a \leq 0.65$, the $\frac{1}{z_{eff}}$ approaches close to the value 0.25 for some intermediate time. But for higher noise strengths $\eta_a \geq 0.75$ the $\frac{1}{z_{eff}}$ remains close to value 0.33 at all times.

And for all noise strengths and at late times the $\frac{1}{z_{eff}}$ is close to 0.33.

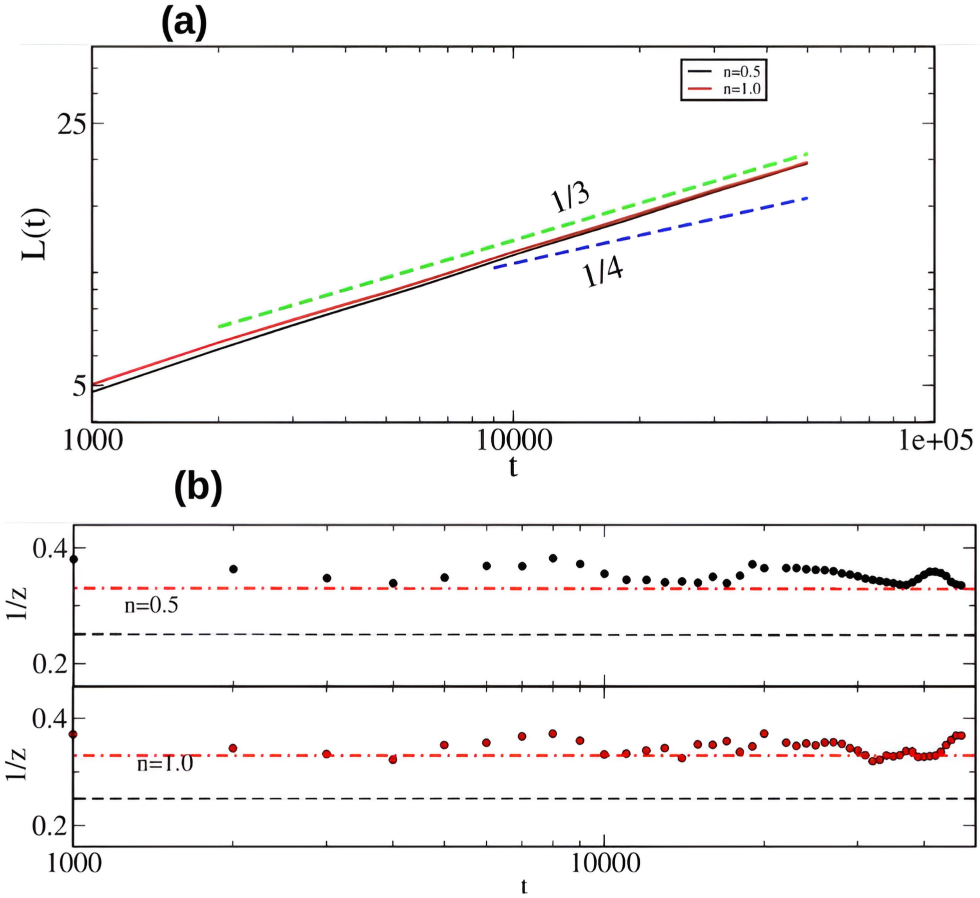

Fig. 5.6 (color online) (a) Characteristic length is shown for $\psi_0 = -0.05$ for two different strengths of noise $\eta_a = 0.5$ and 1.0. Activity, λ is fixed to 0.5. Green and blue dashed lines are lines of slopes $1/3$ and $1/4$ respectively. (b) Shows the effective growth exponent for $\psi_0 = -0.05$ for noise strengths $\eta_a = 0.5$ and 1.0. Activity, λ is fixed to 0.5. Black and red dashed lines show the value 0.25 and 0.33 respectively

Till now we have discussed the results when λ and ψ_0 are of same sign, i.e. $\lambda\psi_0 > 0$. In the next section, we discuss the case, when they are opposite in sign, i.e. $\lambda\psi_0 < 0$

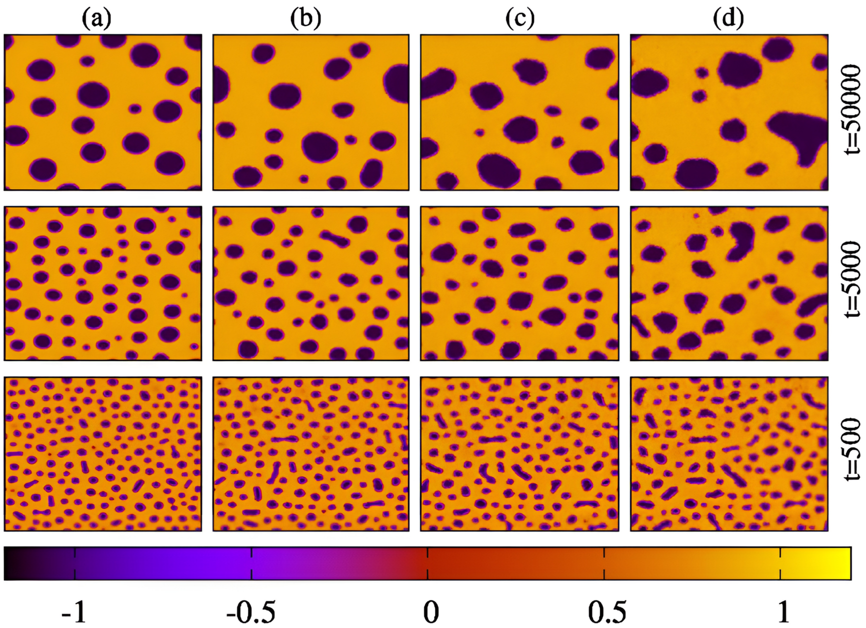

Fig. 5.7 (color online) Time evolution of the system for different values of noise strengths η_a, keeping the activity, λ fixed to 0.5. (a),(b),(c) and (d) are for noise strengths 0.2, 0.5, 0.8 and 1.0 respectively. The average order parameter value is $\psi_0 = 0.4$. The color bar shows the range of ψ value.

5.2.2 Slightly off-critical composition (opposite sign, $\lambda\psi_0 < 0$)

Domain morphology and growth kinetics

We start analysing the results first by observing the domain structure with time. We plot the time evolution snapshots, shown in 5.4. We find that domains get elongated as we increase the noise strengths. Also we show the comparison plots of the snapshots for both the cases in fig 5.5. We find that for the same sign, the domains are elongated for smaller noise strength and become bi-continuous for higher strength of noise, $\eta_a > 0.5$, whereas for opposite sign, the structures of domains are isolated for smaller noise strength and elongated for noise strength, $\eta_a > 0.5$. Next we observe the growth length and exponent for two strengths of noise, moderate $\eta_a = 0.5$ and high $\eta_a = 1.0$, shown in fig 5.6 leads us to know that opposite sign of λ and η_a nullify the effect of each other and effective growth exponent never crosses the value 0.33. It always remains close to 0.33 at all times

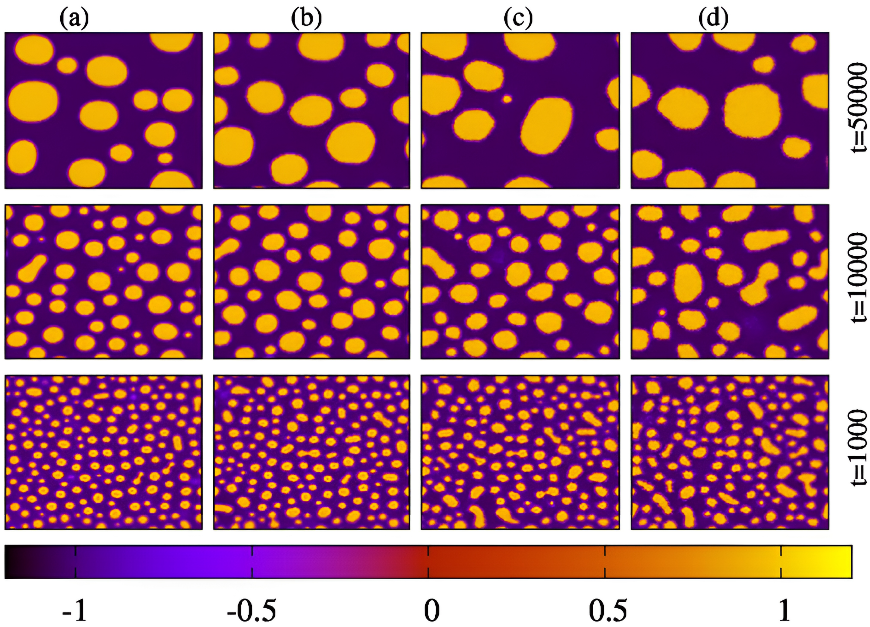

Fig. 5.8 (color online) Time evolution of the system for different values of noise strengths η_a, keeping the activity, λ fixed to 0.5. (a),(b),(c) and (d) are for noise strengths 0.2, 0.5, 0.8 and 1.0 respectively. The average order parameter value is $\psi_0 = -0.4$. The color bar shows the range of ψ value.

and each noise strength, whereas for same sign of λ and ψ_0, for moderate η_a, the growth exponent approaches to 0.25 for some intermediate time as shown in fig 5.5.

5.2.3 Deep off-critical composition

Domain morphology and growth kinetics

Till now we have analysed the results when the system is slightly away from the critical mixture ($\psi_0 = \pm 0.05$). In this section we discuss about the composition far away from critical mixture, i.e one component in the system is quite less in fraction in comparison to the other. We have shown the domain structures for $\psi_0 = \pm 0.4$ in fig 5.7 and 5.8 respectively. We vary the noise strength and observe that initially there are droplets type structures of the particles which are low in fraction with the high fraction of the particles in the background in respective cases and these structures evolve with time in each case.

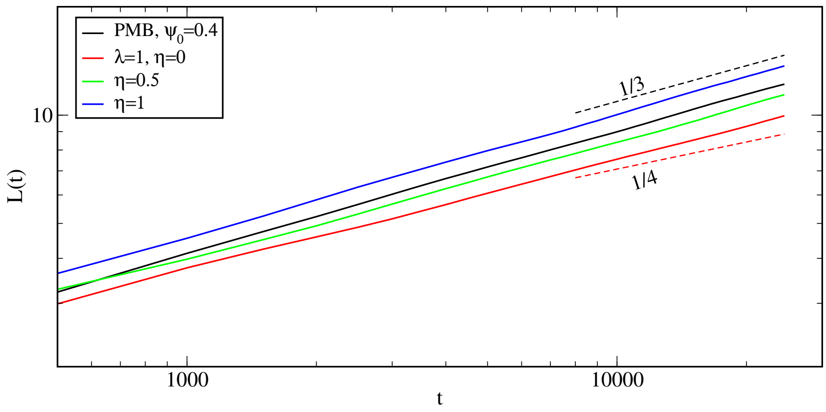

Fig. 5.9 (color online) Characteristic length is shown for $\psi_0 = 0.4$ for two different strengths of noise $\eta_a = 0.5$ and 1.0. Activity, λ is fixed to 1. Black and red dashed lines are lines of slopes $1/4$ and $1/3$ respectively.

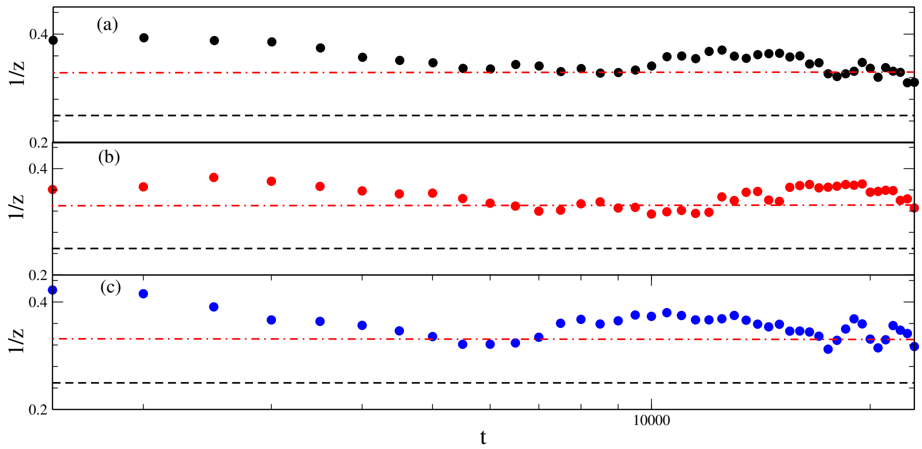

Fig. 5.10 (color online) Shows the effective growth exponent for $\psi_0 = 0.4$ for PMB (a) and for AMBN with noise strengths $\eta_a = 0.5$ and 1.0, (b) and (c) respectively. Activity, λ is fixed to 1.0. Black and red dashed lines show the value 0.25 and 0.33 respectively.

We do not observe any significant difference due to asymmetry present in the system with respect to the sign of activity, λ and off-criticality ψ_0.

We quantify the growth in the case of deep off-criical composition also, where we find that exponent always approaches to the value 0.33 irrespective to the value of noise strength.

5.3 Conclusion

We have performed the study for the growth kinetics of AMB in presence of multiplicative noise for off-critical composition. We observe the two cases (i) slightly off- critical and (ii) deep off-critical mixture. We summarize our results as follows. First we have seen the domain structures and growth exponent for slightly off-critical composition, where we find that the exponent has a significant effect of relative sign of activity and off-criticality. When both are same in sign, the activity, λ plays a role for moderate noise strength and growth approaches to the value 0.25 for some intermediate time. For high noise strength, the exponent is always 0.33. But when activity and off-criticality are opposite in sign, there is no effect of activity, λ on the domain growth and growth exponent is always close to 0.33, irrespective to the strength of noise. For deep off-critical mixture, we find no such dependence of growth kinetics on the relative sign of activity and off-criticality and the exponent is always 0.33.

****