

---

---

## CERTIFICATE


---

---

This is to certified that the work contained in the thesis titled "*INVESTIGATION ON EFFECTIVE PROPERTIES AND PERFORMANCE PARAMETERS OF PIEZOELECTRIC COMPOSITES*" by "*SANJEEV KUMAR SINGH*" has been carried out under my/our supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. Degree.

Supervisor

  
29-11-2021

Prof. S. K. Panda

Department of Mechanical Engineering  
Indian Institute of Technology  
(Banaras Hindu University)  
Varanasi – 221 005, INDIA

Prof. S. K. Panda  
Department of Mechanical Engineering  
Indian Institute of Technology  
(Banaras Hindu University)  
Varanasi – 221 005, INDIA

---

---

## DECLARATION BY THE CANDIDATE


---

---

I "**Sanjeev Kumar Singh**", certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of "**Prof. S. K. Panda**" from "**DECEMBER 2013** to "**DECEMBER 2021**", at the "**DEPARTMENT OF MECHANICAL ENGINEERING**", Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma.

I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

Date: 29/11/2021  
Place: Varanasi

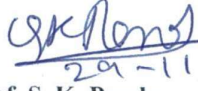
  
Sanjeev Kumar Singh

### CERTIFICATE FROM THE SUPERVISOR

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

  
**Professor and Head**  
Department of Mechanical Engineering  
Indian Institute of Technology  
(Banaras Hindu University)  
Varanasi

विभागाध्यक्ष / HEAD  
यांत्रिक अभियान्तिकी विभाग / Deptt. of Mechanical Engg.  
भारतीय प्रौद्योगिकी संस्थान / Indian Institute of Technology  
(का०हि०वि० / B.H.U.)  
वाराणसी-221005 / Varanasi, India

**Supervisor**  
  
29-11-2021  
**Prof. S. K. Panda**  
Department of Mechanical Engineering  
Indian Institute of Technology  
(Banaras Hindu University)  
Varanasi

विभागाध्यक्ष / HEAD  
यांत्रिक अभियान्तिकी विभाग / Deptt. of Mechanical Engg.  
भारतीय प्रौद्योगिकी संस्थान / Indian Institute of Technology  
(का०हि०वि० / B.H.U.)  
वाराणसी-221005 / Varanasi, India

---

---

## COPY RIGHT TRANSFER CERTIFICATE

---

---

**Title of the Thesis: INVESTIGATION ON EFFECTIVE PROPERTIES AND  
PERFORMANCE PARAMETERS OF PIEZOELECTRIC COMPOSITES**

**Name of the Student: SANJEEV KUMAR SINGH**

### Copyright Transfer

The undersigned hereby assigns to the Indian Institute of Technology (Banaras Hindu University) Varanasi all rights under copyright that may exist in and for the above thesis submitted for the award of the "*DOCTOR OF PHILOSOPHY*".

Date: 29/11/2021

Place: Varanasi



(SANJEEV KUMAR SINGH)

**Note: However, the author may reproduce or authorize others to reproduce material extracted verbatim from the thesis or derivative of the thesis for author's personal use provided that the source and the Institute's copyright notice are indicated.**

## **Abstract**

Over the past three decades, engineers have developed piezoelectric composite materials that enable them to tailor the effective electromechanical properties for a specific application. These materials are manufactured by combining piezoelectric ceramics with piezoelectrically active/passive polymers in a variety of geometrical configurations. As it happens with any composite material, the properties and behaviour of piezocomposites are highly dependent on the properties of the constituent materials and the local arrangement of the different phases. In particular, the physics of the interface plays an important role in determining the electromechanical coupling in the piezocomposite. In the present thesis, the electromechanical behaviour of the piezocomposite is investigated from both theoretical and numerical stand points followed by the validation of the performance parameters of such piezocomposites with the experimental results.

Theoretical investigations are centered on the development of a micromechanics model for predicting the local fields and effective behaviour in the piezo-composites. Extending the results of the constraint tensors, namely Eshelby tensors obtained earlier for the elastic isotropic inclusion cases; a set of four constraint tensors analogous to the Eshelby tensors are developed for mapping the electromechanical coupled field responses in a piezoelectric composite. The interaction among inclusions or inhomogeneities (in the case of piezocomposites) are approximated through the Mori-Tanaka mean field approach. The final relation to determine effective coefficients of the bulk piezocomposite is then derived with the equivalent inclusion method.

The derived micromechanics model based on electro-elastic Eshelby tensors are used to study various aspects of piezoelectric composites. The derived exact results for

electro-elastic Green's function are plotted to understand how the coupled responses are distributed in the space. The results suggest that the distribution of all the Green's functions are symmetric around the origin in the three-dimensional space, while the order of their magnitude varies significantly. In particular, the electric potential induced as a response to a unit electric charge and a unit force differ by 12 orders of magnitude, while the difference between electric potentials differs by only 2 orders of magnitude.

The formula to predict the effective coefficients of the piezoelectric composites developed earlier are utilized to study the behaviour of these coefficients as a function of aspect ratio and volume fraction of the fiber. From the material system studied, it is observed that the longitudinal elastic coefficients and the piezoelectric constants decrease with increasing fiber aspect ratio; in-plane shear modulus remains independent of the fiber aspect ratio; and the dielectric coefficients uniformly increase with the fiber aspect ratio. It is also noticed that, the predicted value of all the coefficients attains a saturation value at fiber aspect ratio of 10 and beyond this ratio the geometry of the inclusion doesn't have significant effects on these coefficients.

For the case of long fiber composites (1-3), a refined model is presented, namely modified strength of materials (MSM), to study the effects of fiber packing geometry and Poisson's ratio mismatch on the effective coefficients of such composites. With the derived parameters of the MSM model another model is developed based on the conservation of strain energy. The predicted results with these proposed models are then compared with the existing strength of material (SM) model. A finite element analysis (FEA) is also carried to validate the results for the six discrete fiber volume fractions (0.1-0.6). Later, the proposed MSM model is utilized to predict performance parameters (or figures of merit) of the piezoelectric composites.

## **Acknowledgements**

I express my sincere thanks and gratitude beyond words to my esteemed supervisor, Prof. S. K. Panda for his consistent help, encouragement and valuable discussions during the entire period of my research work. It would not have been possible to complete the thesis without his utmost involvement and invaluable efforts. He motivated me to pursue research problem and the need for persistent effort to accomplish the goal. I am truly indebted to him.

Besides my supervisor, I would like to thank my RPEC members, Prof. P. Maiti of School of Materials Science & Technology and Dr. R. K. Gautam of Department of Mechanical Engineering for their insightful comments and encouragement. I sincerely thank Prof. Santosh Kumar, Head of the Department of Mechanical Engineering and all the former heads for providing all the research facilities to successfully accomplish my research in the department. I have deep sense of gratitude to all the faculty members especially, Prof. Sanjay Kumar Sinha, Prof. Rajesh Kumar, Prof. Sandeep Kumar and Dr. Amit Tyagi of the Department of Mechanical Engineering, IIT (BHU), Varanasi for their cooperation and inspiration.

I am thankful to all to all my friends: Arun Kumar, Vaibhav Pandey, Vishwas Acharya, Vipul Saxena, Raghunath Prasad, Param Srivastava, Ashish Singh Pareta, Praveen Kumar Singh, Anupam Tiwari, Aishwarya Sheel Wali, Narendra Kumar Jha and seniors especially Nishant Singh and Manvandra Singh for their constant encouragement and for being with me in my moments of happiness and troubles at IIT (BHU), Varanasi.

I would like to express my deepest gratitude to my parents Mrs. Ranju Singh and Mr. Vinod Kumar Singh for their unconditional support and encouragement to pursue my interest. I would also like to thank my brother Mr. Amit Singh for his consistent support and encouragement throughout this journey.

Last but not the least, I wish to thank my friends and the persons whose names have not been mentioned on this piece of paper for extending their cooperation directly or indirectly.

**Sanjeev Kumar Singh**

## Contents

<b>Abstract</b> .....	<b>v-vi</b>
<b>Acknowledgements</b> .....	<b>vii-viii</b>
<b>Contents</b> .....	<b>ix-xii</b>
<b>List of Figures</b> .....	<b>xiii-xix</b>
<b>List of Tables</b> .....	<b>xxi</b>
<b>Abbreviations</b> .....	<b>xxiii</b>
<b>Nomenclature</b> .....	<b>xxv-xxvii</b>
<b>Preface</b> .....	<b>xxix-xxxii</b>
<b>Chapter 1 Introduction</b> .....	<b>1-14</b>
1.1. Piezoelectricity.....	1
1.2. Piezoelectric Effect- Basic Mathematical Formulation.....	5
1.3. Piezoelectric Composites.....	7
<i>1.3.1. Background</i> .....	7
<i>1.3.2. Connectivity Patterns</i> .....	8
<i>1.3.3. Composite Effects</i> .....	11
1.4. Motivation.....	13
1.5 Summary.....	14
<b>Chapter 2 Literature Review</b> .....	<b>15-31</b>
2.1. Analytical Models.....	15
2.2. Numerical Approaches.....	19
2.3. Experimental Characterization.....	24
2.4. Objective of the Present Thesis.....	29
<b>Chapter 3 Micromechanics of Piezoelectric Inclusion and Inhomogeneities</b> ...33-76	
3.1. Introduction.....	33

3.2. Governing Equations of Piezoelectricity .....	35
3.3. Piezoelectric Inclusion and Inhomogeneities .....	38
3.3.1. <i>Evaluation of Electroelastic Green's Functions</i> .....	45
3.3.2. <i>Electroelastic Eshelby Tensors for Spheroidal Inclusion</i> .....	49
3.4. Effective Electroelastic Moduli .....	54
3.4.1. <i>Traction-electric displacement prescribed</i> .....	54
3.4.2. <i>Elastic displacement-electric field prescribed</i> .....	58
3.5. Results and Discussion .....	60
3.6. Summary .....	76
<b>Chapter 4 Effective Properties of Short-fiber Piezoelectric Composites.....</b>	<b>77-105</b>
4.1. Introduction .....	77
4.2. Coupled Electro-Elastic Fields Modeling inside an Inclusion .....	79
4.3. Calculation of Effective Properties .....	84
4.4. Finite Element Modeling .....	87
4.4.1. <i>Calculation of Effective Properties through FEM</i> .....	90
4.4.2. <i>Boundary Conditions and Parameter Calculations</i> .....	92
4.5. Results and Discussion .....	97
4.6. Summary .....	104
<b>Chapter 5 Effective Properties of Long-fiber Piezoelectric Composites.....</b>	<b>107-156</b>
5.1. Introduction .....	107
5.2. Constitutive Relations .....	110
5.3. Micromechanics Model .....	114
5.3.1. <i>Modified Strength of Materials Method (MSM) Model</i> .....	114
5.3.2. <i>Strain Energy Method</i> .....	122
5.3.3. <i>Results and Discussion</i> .....	126

5.4. Numerical Model.....	136
5.4.1. Finite Element Method (FEM).....	136
5.4.2. Periodic Boundary Conditions.....	137
5.4.3. Calculation of Effective Coefficients through FEM.....	138
5.4.4. Results and Discussion.....	146
5.5. Summary.....	154
<b>Chapter 6 Effective Performance Parameters of Piezoelectric Composites.....</b>	<b>157-182</b>
6.1. Introduction.....	157
6.2. Governing Equation of Piezoelectricity.....	161
6.3. Modified Strength of Materials (MSM) Model.....	162
6.4. Analytical Model base on Mori-Tanaka Approach.....	166
6.5. Performance Parameters of 1-3 Piezocomposite.....	168
6.5.1. Piezoelectric Charge Coefficient.....	168
6.5.2. Hydrostatic Electromechanical Coupling Factor.....	169
6.5.3. Electromechanical Coupling Constant.....	169
6.5.4. The Acoustic Impedance.....	170
6.5.5. Stiffened Longitudinal Velocity.....	170
6.6. Results and Discussion.....	171
6.7. Summary.....	181
<b>Chapter 7 Conclusions and Suggestions for Future Work.....</b>	<b>183-186</b>
7.1. General Conclusions.....	183
7.2. Suggestions for Future Work.....	185
<b>References.....</b>	<b>187-204</b>
<b>Appendix A.....</b>	<b>205</b>

<b>Appendix B.....</b>	<b>207-208</b>
<b>Appendix C.....</b>	<b>209-210</b>
<b>List of Publications.....</b>	<b>211</b>

## List of Figures

Figure 1.1 Piezoelectricity: An intermingling of elastic and electric phenomenon.....	1
Figure 1.2 Piezoelectric materials in sensing and actuating applications; (a) P-E hysteresis plot (top) and S-E plot (bottom); (b) The piezoelectric material before and after poling; (c) Change in dimension when applied voltage has polarity similar to that of poling voltage; (d) Change in dimension when the applied voltage has polarity opposite to that of poling voltage; (e) The voltage generated equivalent to poling voltage when compressive force is applied in poling direction; (f) The voltage generated equivalent to poling voltage when tensile force is applied in poling direction.....	4
Figure 1.3 Connectivity families for diphasic composites. The total number of connectivity patterns arising out of 10 families depicted is 16 due to permutations of order involved in families; {0-2}, {0-3}, {1-0}, {1-3}, {2-1}, {2-3}.....	9
Figure 1.4 Schematic diagram showing composites with various connectivity patterns realized over the past 40 years.....	10
Figure 1.5 Composite effects: (a) sum effect; (b) combination effect;(c) product effect..	12
Figure 3.1(a) Schematic of an ellipsoidal inclusion in a matrix medium.....	39
Figure 3.1(b) Schematic of a spheroidal inclusion in a matrix medium.....	50
Figure 3.2 The distribution of the Green's function $G_{11}$ in space.....	61
Figure 3.3 The distribution of the Green's function $G_{12}$ in space.....	62
Figure 3.4 The distribution of the Green's function $G_{13}$ in space.....	62
Figure 3.5 The distribution of the Green's function $G_{14}$ in space.....	63
Figure 3.6 The distribution of the Green's function $G_{22}$ in space.....	63

Figure 3.7 The distribution of the Green's function $G_{23}$ in space.....	64
Figure 3.8 The distribution of the Green's function $G_{24}$ in space.....	64
Figure 3.9 The distribution of the Green's function $G_{33}$ in space.....	65
Figure 3.10 The distribution of the Green's function $G_{34}$ in space.....	65
Figure 3.11 The distribution of the Green's function $G_{44}$ in space.....	66
Figure 3.12 Vanishing Eshelby tensors at infinite aspect ratio.....	67
Figure 3.13 Non-vanishing Eshelby tensors at infinite aspect ratio.....	68
Figure 3.14 Eshelby tensors involving interaction between elastic and electric fields....	68
Figure 3.15 The composite normal modulus $E_{1111}$ against fiber volume fraction for various aspect ratios, $\alpha_f$ .....	70
Figure 3.16 The composite normal modulus $E_{1122}$ against fiber volume fraction for various aspect ratios, $\alpha_f$ .....	70
Figure 3.17 The composite normal modulus $E_{1133}$ against fiber volume fraction for various aspect ratios, $\alpha_f$ .....	71
Figure 3.18 The composite normal modulus $E_{3333}$ against fiber volume fraction for various aspect ratios, $\alpha_f$ .....	71
Figure 3.19 The composite shear modulus $E_{2323}$ against fiber volume fraction for various aspect ratios, $\alpha_f$ .....	72
Figure 3.20 The composite shear modulus $E_{1212}$ against fiber volume fraction for various aspect ratios, $\alpha_f$ .....	72
Figure 3.21 The composite piezoelectric constant $E_{4311}$ against fiber volume fraction for various aspect ratios, $\alpha_f$ .....	73
Figure 3.22 The composite piezoelectric constant $E_{4333}$ against fiber volume fraction for various aspect ratios, $\alpha_f$ .....	73

Figure 3.23 The composite dielectric constant $E_{4141}$ against fiber volume fraction for various aspect ratios, $\alpha_f$ .....	74
Figure 3.24 The composite dielectric constant $E_{4343}$ against fiber volume fraction for various aspect ratios, $\alpha_f$ .....	74
Figure 4.1(a) FEA model of an active composite containing spheroid inclusion; (i) Structure of composite (mesh element view); (ii) RVE of SC fiber arrangement (inner inclusion view); (iii) RVE of BCC fiber arrangement (inner inclusion view).....	91
Figure 4.1(b) The convergence graph for determining ideal mesh size for the FE Analysis.....	92
Figure 4.2 Representative Volume Element (RVE) depicting von Mises stress distribution containing spheroid inclusion in SC arrangement (Fiber volume fraction ranging from 0.1-0.5), (a) 0.1, (b) 0.2, (c) 0.3, (d) 0.4, (e) 0.5.....	95
Figure 4.3 Representative Volume Element (RVE) depicting von Mises stress distribution containing spheroid inclusion in BCC arrangement (Fiber volume fraction ranging from 0.1-0.5), (a) 0.1, (b) 0.2, (c) 0.3, (d) 0.4, (e) 0.5.....	96
Figure 4.4 Comparison of the predicted effective elastic coefficient $C_{11}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	97
Figure 4.5 Comparison of the predicted effective elastic coefficient $C_{12}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	98
Figure 4.6 Comparison of the predicted effective elastic coefficient $C_{13}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	98

Figure 4.7 Comparison of the predicted effective elastic coefficient $C_{33}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	99
Figure 4.8 Comparison of the predicted effective elastic coefficient $C_{44}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	99
Figure 4.9 Comparison of the predicted effective elastic coefficient $C_{66}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	100
Figure 4.10 Comparison of the predicted effective piezoelectric coefficient $e_{31}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	101
Figure 4.11 Comparison of the predicted effective piezoelectric coefficient $e_{33}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	101
Figure 4.12 Comparison of the predicted effective piezoelectric coefficient $e_{15}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	102
Figure 4.13 Comparison of the predicted effective dielectric coefficient $\kappa_{11}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	103
Figure 4.14 Comparison of the predicted effective dielectric coefficient $\kappa_{33}$ of BaTiO <sub>3</sub> /PZT-5H composite with respect to fiber volume fraction for the analytical Eshelby Method and numerical FEM Method.....	103
Figure 5.1 (a) Schematic of a piezoelectric fiber reinforced composite PFRC (1-3	

piezocomposite).....	116
Figure 5.1 (b) A representative volume element (RVE).....	116
Figure 5.2 Transverse cross-section of a representative volume element of PFRC (1-3 piezocomposite).....	117
Figure 5.3 The effective elastic coefficient $C_{11}$ as predicted by the model developed in the present study (MSM), SM Model and Energy Model with change in fiber volume fraction.....	127
Figure 5.4 The effective elastic coefficient $C_{12}$ as predicted by the model developed in the present study (MSM), SM Model and Energy Model with change in fiber volume fraction.....	128
Figure 5.5 The effective elastic coefficient $C_{13}$ as predicted by the model developed in the present study (MSM), SM Model and Energy Model with change in fiber volume fraction.....	128
Figure 5.6 The effective elastic coefficient $C_{33}$ as predicted by the model developed in the present study (MSM), SM Model and Energy Model with change in fiber volume fraction.....	130
Figure 5.7 Comparison of predicted parameter $R_{31}$ with change in fiber volume fractions.....	131
Figure 5.8 Comparison of predicted parameter $R_{32}$ with change in fiber volume fractions.....	132
Figure 5.9 Comparison of predicted parameter $R_{33}$ with change in fiber volume fractions.....	132
Figure 5.10 Comparison of predicted effective dielectric constant $\kappa_{33}$ with change in fiber volume fractions.....	135
Figure 5.11 The convergence graph for determining ideal mesh size for the FE	

analysis.....	139
Figure 5.12(a) RVE of finite element analysis (FEA): Fiber volume fractions (0.1-0.6 with step size of 0.1) taken to evaluate effective properties.....	144
Figure 5.12(b) The distribution of stress/electric field in the RVE when the mechanical load (in z-direction) and boundary conditions are applied; (i) stress in direction-1 (x-axis); (ii) stress in direction-2 (y-axis); (iii) stress in direction-3 (z-axis); (iv) electric field distribution in direction-3 (z-axis).....	145
Figure 5.13 The predicted effective elastic coefficient $C_{11}^c$ with change in fiber volume fraction estimated by strength of materials model, present micromechanics model and FEM model.....	149
Figure 5.14 The predicted effective elastic coefficient $C_{12}^c$ with change in fiber volume fraction estimated by strength of materials model, present micromechanics model and FEM model.....	149
Figure 5.15 The predicted effective elastic coefficient $C_{13}^c$ with change in fiber volume fraction estimated through strength of materials model, present micromechanics model and FEM model.....	150
Figure 5.16 The predicted effective elastic coefficient $C_{33}^c$ with change in fiber volume fraction estimated by strength of materials model, present micromechanics model and FEM model.....	150
Figure 5.17 The predicted effective piezoelectric coefficient $e_{31}^c$ with change in fiber volume fraction estimated by strength of materials model, present micromechanics model and FEM model.....	152
Figure 5.18 The predicted effective piezoelectric coefficient $e_{33}^c$ with change in fiber volume fraction estimated by strength of materials model, present	

micromechanics model and FEM model.....	152
Figure 5.19 The predicted effective dielectric constant $\kappa_{33}^c$ with change in fiber volume fraction estimated by strength of materials model, present micromechanics model and FEM model.....	153
Figure 6.1 Schematic of a 1-3 piezoelectric composite.....	163
Figure 6.2 Transverse cross-section of a representative volume element (RVE) of 1-3 piezoelectric composite.....	164
Figure 6.3 Comparison of effective electromechanical coupling constant $K_I$ of 1-3 piezoelectric composite between experimental measurements and present analysis as a function of fiber volume fraction.....	173
Figure 6.4 Comparison of effective acoustic impedance $Z$ of 1-3 piezoelectric composite between experimental measurements and present analysis as a function of fiber volume fraction.....	174
Figure 6.5 Comparison of effective stiffened longitudinal velocity $V_{33}^D$ of 1-3 piezoelectric composite between experimental measurements and present analysis as a function of fiber volume fraction.....	176
Figure 6.6 Comparison of effective short-circuit stiffness constant $C_{33}^D$ of 1-3 piezoelectric composite between experimental measurements and present analysis as a function of fiber volume fraction.....	178
Figure 6.7 Comparison of effective short-circuit stiffness constant $C_{33}^E$ of 1-3 piezoelectric composite between experimental measurements and present analysis as a function of fiber volume fraction.....	179
Figure 6.8 Comparison of effective hydrostatic charge coefficient $d_h$ of 1-3 piezoelectric composite between experimental measurements and present analysis as a function of fiber volume fraction.....	180

## **List of Tables**

Table 2.1 A summary of the analytical models and numerical models developed to predict effective coefficients of piezocomposites.....	25
Table 3.1 Electroelastic constants of the constituent materials.....	61
Table 4.1 Material properties of fiber and matrix phases.....	89
Table 5.1. Material properties of fiber and matrix phases.....	126
Table 5.2: Material properties of the fiber and matrix phases for FEM calculations.....	140
Table 5.3 List of boundary conditions applied on RVE and formula for calculation of effective coefficients.....	142
Table 5.4 Classifications of coefficients in groups according to the boundary conditions used for FEM analysis.....	146
Table 6.1 Material properties of constituents (fiber and matrix phases).....	172

## **Abbreviations**

BCC	Body Centered Cubic
EM	Eshelby Method
FEA	Finite Element Analysis
FEM	Finite Element Method
GF	Green's Function
MSM	Modified Strength of Materials Model
MT	Mori-Tanaka Approximation
SC	Simple Cubic
SM	Strength of Materials Model

## Nomenclature

Symbol	Description
$\Omega$	Domain of the inclusion
$D$	Entire composite domain
$a_1, a_2, a_3$	Lengths of the semi axes of the ellipsoid
$\varepsilon_{mn}^*$	Eigen strain
$E_n^*$	Eigen electric field
$Z_{mn}^*$	Eigen strain and electric field
$C_{ijmn}$	Elastic stiffness
$e_{nij}$	Piezoelectric stress coefficients
$\kappa_{in}$	Dielectric constants
$\varepsilon_{mn}$	Strain
$E_n$	Electric field
$u_i, u_j$	Elastic displacement
$\phi$	Electric potential
$\sigma_{ij}$	Stress
$D_i$	Electric displacement
$U_M$	Elastic displacement and electric potential
$Z_{Mn}$	Strain and electric field
$\Sigma_{iJ}$	Stress and electric displacement
$L_{iJMn}$ or $E_{iJMn}$	Electro-elastic constants of the material
$N_{MJ}(\xi)$	Cofactors of $4 \times 4$ matrix $L_{iMJn} \xi_i \xi_n$
$D(\xi)$	Determinants of $4 \times 4$ matrix $L_{iMJn} \xi_i \xi_n$

$G_{Mjm}(x-x')$	Coupled electro-elastic Green's function
$\xi_1, \xi_2, \xi_3$	Coordinate transformation variables
$\zeta_1, \zeta_2, \zeta_3$	Coordinate transformation variables
$S_{MnAb}$	Coupled electro-elastic Eshelby tensors
$L_{ijMn}^*$ or $E_{ijMn}^*$	Electro-elastic constants within an inclusion
$\Sigma_{ij}^0$	Stress and dielectric displacement in the absence of inhomogeneities
$Z_{Mn}^0$	Strain and electric field in the absence of inhomogeneities
$n_i$	An outward normal vector to the surface of the inclusion
$\Sigma_{ij}^m$	Stress and electric displacement in the matrix medium
$Z_{ij}^m$	Strain and electric field in the matrix medium
$\Sigma_{ij}^\Omega$	Stress and dielectric displacement in any arbitrary inhomogeneity
$I_{MnAb}$	An identity matrix of order nine.
$f$	Fiber volume fraction
$V$	Volume of a representative volume element (RVE)
$\langle T_{ij} \rangle$	Average spatial value of stress in FEA
$\langle S_{ij} \rangle$	Average spatial value of strain in FEA
$C_{ijkl}^{eff}$	Effective elastic constants in FEA

$e_{ikl}^{eff}$	Effective piezoelectric constants in FEA
$\kappa_{ik}^{eff}$	Effective dielectric constants in FEA
$\langle \dots \rangle$	Average of directional field quantities in FEA

**Co-ordinates:**

1, 2, 3	Local material cartesian coordinates
$x, y, z$	Local cartesian coordinate system in FEA
$x_1, x_2, x_3$	Local cartesian coordinate system in long fiber (1-3) piezocomposite analysis
$X, Y, Z$	Global cartesian coordinate system

## Preface

The effective properties of piezoelectric composites have attracted much attention because of technological applications and theoretical interest. The applications of effective piezoelectric properties include the ultrasonic transducer of underwater acoustics, biomedical imaging, electronic instrumentation, etc. To investigate the effective response mechanism of piezoelectric composites, recently many authors proposed methods to estimate the effective properties of piezoelectric composites. The methods to predict these response parameters involve a number of theoretical investigations from simple approximation methods to effective medium approximations. Few other theoretical models involve polarization field method, virtual work theorems, asymptotic homogenization techniques, transformation field method, Mori-Tanaka approximations etc. However, there is not a useful method for estimating the effective responses of the anisotropic piezoelectric composites having complex shape of inclusions, and hence up to now, the effective response mechanism could not be explained till now.

From all the established theories it's evident that the effective piezoelectric properties are not only related to the physical properties of the materials and volume fraction of inclusions; but also, to the shapes of the inclusion materials. Different shapes of inclusions induce different internal strain and electric fields, and then these induced fields result in the different effective piezoelectric tensors. Furthermore, it's also learnt through these properties that the effective dielectric or elastic response may be affected by the elastic or dielectric properties of composites. Therefore, it becomes a dire need to develop a method for predicting the effective response of piezoelectric composites with arbitrary geometric structure of inclusions and investigating their effective response mechanism. This thesis presents an effort to study such long standing problem related to

prediction of electroelastic response of piezoelectric composites with theoretical and numerical standpoint.

Chapter 1 presents an introduction to the concept of piezoelectricity and underlying mathematical relations that define piezoelectric effects. It also presents a discussion of how these piezoelectric materials are connected to other active/passive materials to form piezoelectric composites and what may be their possible combining effects on the overall composite properties.

Chapter 2 presents an extensive literature review of the analytical, numerical and experimental approaches developed over the years for the determination of effective properties of piezo composites. It also presents a discussion about the gaps within the literature and outlines the objectives to carry out the present research.

Chapter 3 presents a comprehensive micromechanics approach to study the electromechanical response of piezoelectric inclusion and inhomogeneities followed by the derivation of expressions to evaluate effective properties of piezocomposites containing short fibers. The effective elastic, piezoelectric and dielectric constants is evaluated with respect to change in fiber aspect ratios. The piezoelectric analogue for Eshelby tensors for elasticity is derived for short-fiber piezocomposites. The precursor to these tensors, i.e., electroelastic Green's functions has been evaluated for the transversely isotropic symmetry condition. The spatial distribution and nature of these functions are also studied in great detail.

Chapter 4 deals with a numerical approach to ascertain the validity of Eshelby model for short-fiber piezocomposite problems. A finite element method-based approach is adopted to simulate the short-fiber piezo inclusion problem, and with applied periodic boundary conditions the effective coefficients for such composites is evaluated. The numerical results for each coefficient are then compared with the results of Eshelby model

to check the validity the model for short-fiber problems.

Chapter 5 provides an analytical model that is based on improved methodology adopted to estimate effective properties of long-fiber piezocomposites for transverse loading cases. This model is capable of capturing the effects of irregularity of geometries in fibers and Poisson's ratio mismatch on the overall properties of the piezocomposites. The model is developed by doing certain modifications to the conventional strength of materials approach that is used for long-fiber problem and the new model is named as modified strength of materials. The strain energy approach and finite element-based analysis is carried out to establish the validity of the proposed analytical model.

Chapter 6 presents a comprehensive study of the performance parameters such as electromechanical coupling factors, stiffened longitudinal velocity, acoustic impedance, hydrostatic charge coefficients etc. that are essential for applications of piezocomposites. These parameters are studied with the proposed micromechanics model and the accuracy and validity of the results is established by comparing with the other analytical models.

Chapter 7 discusses the general conclusions of the research carried out in this thesis and key findings of the study are elaborated in detail.