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Sanjeev Kumar Singh

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Abstract

Over the past three decades, engineers have developed piezoelectric composite materials that enable them to tailor the effective electromechanical properties for a specific application. These materials are manufactured by combining piezoelectric ceramics with piezoelectrically active/passive polymers in a variety of geometrical configurations. As it happens with any composite material, the properties and behaviour of piezocomposites are highly dependent on the properties of the constituent materials and the local arrangement of the different phases. In particular, the physics of the interface plays an important role in determining the electromechanical coupling in the piezocomposite. In the present thesis, the electromechanical behaviour of the piezocomposite is investigated from both theoretical and numerical stand points followed by the validation of the performance parameters of such piezocomposites with the experimental results.

Theoretical investigations are centered on the development of a micromechanics model for predicting the local fields and effective behaviour in the piezo-composites. Extending the results of the constraint tensors, namely Eshelby tensors obtained earlier for the elastic isotropic inclusion cases; a set of four constraint tensors analogous to the Eshelby tensors are developed for mapping the electromechanical coupled field responses in a piezoelectric composite. The interaction among inclusions or inhomogeneities (in the case of piezocomposites) are approximated through the Mori-Tanaka mean field approach. The final relation to determine effective coefficients of the bulk piezocomposite is then derived with the equivalent inclusion method.

The derived micromechanics model based on electro-elastic Eshelby tensors are used to study various aspects of piezoelectric composites. The derived exact results for

electro-elastic Green's function are plotted to understand how the coupled responses are distributed in the space. The results suggest that the distribution of all the Green's functions are symmetric around the origin in the three-dimensional space, while the order of their magnitude varies significantly. In particular, the electric potential induced as a response to a unit electric charge and a unit force differ by 12 orders of magnitude, while the difference between electric potentials differs by only 2 orders of magnitude.

The formula to predict the effective coefficients of the piezoelectric composites developed earlier are utilized to study the behaviour of these coefficients as a function of aspect ratio and volume fraction of the fiber. From the material system studied, it is observed that the longitudinal elastic coefficients and the piezoelectric constants decrease with increasing fiber aspect ratio; in-plane shear modulus remains independent of the fiber aspect ratio; and the dielectric coefficients uniformly increase with the fiber aspect ratio. It is also noticed that, the predicted value of all the coefficients attains a saturation value at fiber aspect ratio of 10 and beyond this ratio the geometry of the inclusion doesn't have significant effects on these coefficients.

For the case of long fiber composites (1-3), a refined model is presented, namely modified strength of materials (MSM), to study the effects of fiber packing geometry and Poisson's ratio mismatch on the effective coefficients of such composites. With the derived parameters of the MSM model another model is developed based on the conservation of strain energy. The predicted results with these proposed models are then compared with the existing strength of material (SM) model. A finite element analysis (FEA) is also carried to validate the results for the six discrete fiber volume fractions (0.1-0.6). Later, the proposed MSM model is utilized to predict performance parameters (or figures of merit) of the piezoelectric composites.

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Sanjeev Kumar Singh

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Abbreviations

BCC	Body Centered Cubic
EM	Eshelby Method
FEA	Finite Element Analysis
FEM	Finite Element Method
GF	Green's Function
MSM	Modified Strength of Materials Model
MT	Mori-Tanaka Approximation
SC	Simple Cubic
SM	Strength of Materials Model

Nomenclature

Symbol	**Description**
Ω	Domain of the inclusion
D	Entire composite domain
$a_1,\ a_2,\ a_3$	Lengths of the semi axes of the ellipsoid
ε_{mn}^{*}	Eigen strain
E_n^{*}	Eigen electric field
Z_{mn}^{*}	Eigen strain and electric field
C_{ijmn}	Elastic stiffness
e_{nij}	Piezoelectric stress coefficients
κ_{in}	Dielectric constants
ε_{mn}	Strain
E_n	Electric field
$u_i\ ,\ u_j$	Elastic displacement
ϕ	Electric potential
σ_{ij}	Stress
D_i	Electric displacement
U_M	Elastic displacement and electric potential
Z_{Mn}	Strain and electric field
Σ_{iJ}	Stress and electric displacement
L_{iJMn} or E_{iJMn}	Electro-elastic constants of the material
$N_{MJ}(\xi)$	Cofactors of 4×4 matrix $L_{iMJn}\xi_i\xi_n$
$D(\xi)$	Determinants of 4×4 matrix $L_{iMJn}\xi_i\xi_n$

$G_{MJin}(x-x')$	Coupled electro-elastic Green's function
ξ_1, ξ_2, ξ_3	Coordinate transformation variables
ζ_1, ζ_2, ζ_3	Coordinate transformation variables
S_{MnAb}	Coupled electro-elastic Eshelby tensors
L^*_{iJMn} or E^*_{iJMn}	Electro-elastic constants within an inclusion
$\Sigma^0{}_{iJ}$	Stress and dielectric displacement in the absence of inhomogeneities
$Z^0{}_{Mn}$	Strain and electric field in the absence of inhomogeneities
n_i	An outward normal vector to the surface of the inclusion
$\Sigma^m{}_{iJ}$	Stress and electric displacement in the matrix medium
$Z^m{}_{iJ}$	Strain and electric field in the matrix medium
$\Sigma^\Omega{}_{iJ}$	Stress and dielectric displacement in any arbitrary inhomogeneity
I_{MnAb}	An identity matrix of order nine.
f	Fiber volume fraction
V	Volume of a representative volume element (RVE)
$\langle T_{ij} \rangle$	Average spatial value of stress in FEA
$\langle S_{ij} \rangle$	Average spatial value of strain in FEA
C^{eff}_{ijkl}	Effective elastic constants in FEA

e_{ikl}^{eff}	Effective piezoelectric constants in FEA
κ_{ik}^{eff}	Effective dielectric constants in FEA
$\langle ... \rangle$	Average of directional field quantities in FEA

Co-ordinates:

$1, 2, 3$	Local material cartesian coordinates
x, y, z	Local cartesian coordinate system in FEA
x_1, x_2, x_3	Local cartesian coordinate system in long fiber (1-3) piezocomposite analysis
X, Y, Z	Global cartesian coordinate system

Preface

The effective properties of piezoelectric composites have attracted much attention because of technological applications and theoretical interest. The applications of effective piezoelectric properties include the ultrasonic transducer of underwater acoustics, biomedical imaging, electronic instrumentation, etc. To investigate the effective response mechanism of piezoelectric composites, recently many authors proposed methods to estimate the effective properties of piezoelectric composites. The methods to predict these response parameters involve a number of theoretical investigations from simple approximation methods to effective medium approximations. Few other theoretical models involve polarization field method, virtual work theorems, asymptotic homogenization techniques, transformation field method, Mori-Tanaka approximations etc. However, there is not a useful method for estimating the effective responses of the anisotropic piezoelectric composites having complex shape of inclusions, and hence up to now, the effective response mechanism could not be explained till now.

From all the established theories it's evident that the effective piezoelectric properties are not only related to the physical properties of the materials and volume fraction of inclusions; but also, to the shapes of the inclusion materials. Different shapes of inclusions induce different internal strain and electric fields, and then these induced fields result in the different effective piezoelectric tensors. Furthermore, it's also learnt through these properties that the effective dielectric or elastic response may be affected by the elastic or dielectric properties of composites. Therefore, it becomes a dire need to develop a method for predicting the effective response of piezoelectric composites with arbitrary geometric structure of inclusions and investigating their effective response mechanism. This thesis presents an effort to study such long standing problem related to prediction of electroelastic response of piezoelectric composites with theoretical and numerical standpoint.

Chapter 1 presents an introduction to the concept of piezoelectricity and underlying mathematical relations that define piezoelectric effects. It also presents a discussion of how these piezoelectric materials are connected to other active/passive materials to form piezoelectric composites and what may be their possible combining effects on the overall composite properties.

Chapter 2 presents an extensive literature review of the analytical, numerical and experimental approaches developed over the years for the determination of effective properties of piezo composites. It also presents a discussion about the gaps within the literature and outlines the objectives to carry out the present research.

Chapter 3 presents a comprehensive micromechanics approach to study the electromechanical response of piezoelectric inclusion and inhomogeneities followed by the derivation of expressions to evaluate effective properties of piezocomposites containing short fibers. The effective elastic, piezoelectric and dielectric constants is evaluated with respect to change in fiber aspect ratios. The piezoelectric analogue for Eshelby tensors for elasticity is derived for short-fiber piezocomposites. The precursor to these tensors, i.e., electroelastic Green’s functions has been evaluated for the transversely isotropic symmetry condition. The spatial distribution and nature of these functions are also studied in great detail.

Chapter 4 deals with a numerical approach to ascertain the validity of Eshelby model for short-fiber piezocomposite problems. A finite element method-based approach is adopted to simulate the short-fiber piezo inclusion problem, and with applied periodic boundary conditions the effective coefficients for such composites is evaluated. The numerical results for each coefficient are then compared with the results of Eshelby model to check the validity the model for short-fiber problems.

Chapter 5 provides an analytical model that is based on improved methodology adopted to estimate effective properties of long-fiber piezocomposites for transverse loading cases. This model is capable of capturing the effects of irregularity of geometries in fibers and Poisson’s ratio mismatch on the overall properties of the piezocomposites. The model is developed by doing certain modifications to the conventional strength of materials approach that is used for long-fiber problem and the new model is named as modified strength of materials. The strain energy approach and finite element-based analysis is carried out to establish the validity of the proposed analytical model.

Chapter 6 presents a comprehensive study of the performance parameters such as electromechanical coupling factors, stiffened longitudinal velocity, acoustic impedance, hydrostatic charge coefficients etc. that are essential for applications of piezocomposites. These parameters are studied with the proposed micromechanics model and the accuracy and validity of the results is established by comparing with the other analytical models.

Chapter 7 discusses the general conclusions of the research carried out in this thesis and key findings of the study are elaborated in detail.