CHAPTER – VI

FREQUENCY DOMAIN BASED FRACTIONAL ORDER PROPORTIONAL INTEGRAL DERIVATIVE CONTROLLER FOR PROCESS CONTROL SYSTEMS

INTRODUCTION

Fractional order systems could model various real materials more adequately than integer order ones and thus provide an excellent modeling tool in describing many actual dynamical processes. As a matter of fact the fractional order systems require the corresponding fractional order controllers to achieve excellent performance. In most cases, however, researchers consider the fractional order controller applied to the integer order plant to enhance the system control performance.

In today’s world, the PID controller is most influential controller in process control systems. Due to its performance effectiveness and easy to implement the proportional integral derivative controller has been widely used in feedback control system. The tuning of proportional integral derivative has gained most of the researcher’s attention since its inception by Ziegler and Nichols in 1942. Specifications, stability, design, applications and performance of the PID controller have been widely treated since then. From last 30 years, it is remarkably and interesting to see that, the application of fractional order controllers has been increased in many areas such as engineering and science. It is only possible due to better understanding of fractional calculus (FC) potentiality revealed by different phenomena such as viscoelasticity, damping, chaos, diffusion and wave propagation, percolation and irreversibility.

In what interests automatic control theory is that the fractional calculus concepts were adapted to frequency-based methods. Manabe et al (1961) presented the applications of fractional order control in both frequency domain and time domain. Oustaloup et al (1999) presented fraction order robust controller called as CRONE (Commande Robuste d’ORdre Non Entier) and proved the superiority in performance over PID controller with the help of simulation results. Podlubny et al (1999) introduced generalized PID controller with integration action λ and derivative action μ. Dynamics of an arbitrary real order are considered while designing the controller. He demonstrated the better response of Fractional PID over Integer Order PID for the fractional order system. Vinagre et al (2000) proposed the tuning of fractional order PID controller by using the frequency domain specifications.

Currently Researchers are devoted their attention in developing new tuning techniques for Fraction order PID controllers. It is important to analyze the behavior of fractional order derivatives and fractional order integrals to develop a effective controller which is used in real life models. Some of these techniques are extension of the classical PID controller technique. Caponetto et al (2002) presents the extension of the differentiation, integration order from integer to non-integer numbers provides a more flexible tuning strategy, and hence it is simple to achieve the desired specifications.

Leu et al (2002) developed the tuning of fractional order PID based on frequency domain specifications like gain crossover frequency and phase margin. He further used optimization technique to minimize integral square error (ISE) in order to calculate the controller's parameters. Sanchez et al (1999) designed the fractional order PID controller and has been applied for active reduction of vertical tail buffering. Calderon et al (2003) developed fractional order PID controller and successfully applied for the buck converter is also called as fractional order sliding mode control. Monje et al (2002) proposed a fractional order controller of the model $D^{\beta}I^{\alpha}$ with fractional order integral and derivative parts. Similarly, he designed fractional order PI^{α} for plants with long dead time. He proved that system controlled with this controller is robust to gain variation i.e., the system obey the iso-dampin property (the open loop phase of the system is flat at gain crossover frequency).

In this chapter, we devoted our work to design the fractional order PID controller by using the frequency domain specification.

$$C_{FOPID}(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{6.1}$$

The main purpose of using this type of fractional order controller is to use the two fractional power parameters to fulfill additional specifications while designing the controller. In conventional integer order PID controller, we have only three parameters i.e. K_p, K_d and K_i. Thus, we can use three control specifications to design the controller. But in the case of the fractional order controller we have 5 parameters and we can fulfill five specifications of the system by designing this type of controller. By using fractional order controller, we can fulfill specifications regarding robustness, noise rejection and disturbance rejection.

Since the designed fractional order PID controller is of irrational function so time domain simulation or implementation will require to band limit its fractional effects. So we are approximation the irrational fractional function into rational function in certain frequency band limit. Further high frequency band limitation of the derivative effects limits its high frequency gain and thus the control effort provide by the controller.

This chapter is organized as follows; section 6.1 summarizes the design specifications of fractional Order PID controller. Design methodology of fractional order PID controller is presented in section 6.2. In section, 6.3 and section 6.5 presents of the proposed fractional order PID controller applied to the system. Results and discussion are shown in section 6.4 and section 6.6. Finally, conclusions are drawn in section 6.7.

6.1 THE DESIGN PROBLEM FORMULATION OF FRACTIONAL ORDER PROPORTIONAL INTEGRAL DERIVATIVE FORMULATED BY THESE FOLLOWING SPECIFICATIONS

As explained above our motto is to design a fractional order PID controller, which has five parameters to tune. So that the closed loop control system can fulfill five different design specifications such as phase margin specification, gain crossover frequency, robustness specification, sensitivity specification and complementary sensitivity function. The reasons for considering these specifications are as they play important role regarding performance, stability and robustness of the control system. Though there are other kind of specifications can also met, depending upon systems requirement.

Hence, the design problem are stated below

- *Phase margin and the gain crossover specification:-*

As discussed earlier the phase margin and the gain crossover frequency are the important parameter which relates the system robustness. Closed loop system damping ratio depends on the phase margin and it can serves as performance measures.

The equations for the gain crossover frequency and the phase margin are given below

$$\left| C(j\omega_{cg})G(j\omega_{cg}) \right|_{dB} = 1dB, \tag{6.2}$$

$$\arg(C(j\omega_{cg})G(j\omega_{cg})) = \pi + \phi_m \tag{6.3}$$

- *Robustness to variation of the gain of the plant:-*

This specification forces the open loop phase of the system to be flat or constant around the gain crossover frequency. It provides the constant open loop phase in spite of the system gain variations and the overshoot of the closed loop response is constant within a gain range also known as iso-damping property of the time response. We have to note that the range of the gain variation is not pre defined for which the system is robust. The designer could not force the system to be robust for particular gain range. It is purely depends on the gain crossover frequency i.e., around the gain crossover frequency the system will be robust.

The equation for robustness is given below

$$\frac{d \arg(F(s))}{d\omega}_{\omega=\omega_{cg}} \tag{6.4}$$

- *Noise rejection at high frequencies:-*

This is the complementary sensitivity function constraint which can be established by the following condition

$$\left| T(j\omega) = \frac{C(j\omega)G(j\omega)}{1+C(j\omega)G(j\omega)} \right|_{dB} \leq AdB \tag{6.5}$$

$$\forall \omega \geq \omega_t \text{ rad/sec} \Rightarrow \left| T(j\omega_t) \right|_{dB} = \text{A dB}$$

Where A dB is the desired attenuation of the noise in the frequency range below ω_t rad/sec.

- *Output disturbance rejection:-*

This is sensitivity function constraint, which can be established by

$$\left| S(j\omega) = \frac{1}{1+C(j\omega)G(j\omega)} \right|_{dB} \leq BdB \tag{6.6}$$

$$\forall \omega \geq \omega_s \text{ rad/sec} \Rightarrow \left| S(j\omega_s) \right|_{dB} = B \text{ dB}$$

Where B dB is the desired value of the sensitivity function in the frequency range below ω_s rad/sec.

Other kind of design specifications can also be met according to the requirement of the system. Using the above mentioned specifications and the different other specifications depending upon the systems requirement, the fractional order PID controller has been designed for a given system. By employing the fractional order PID we can fulfil more specification as compared to the integer order PID controller.

6.2 DESIGN OF FRACTIONAL ORDER PROPORTIONAL INTEGRAL DERIVATIVE CONTROLLER

The structure of FOPID controller considered here is in the form

$$C_{FOPID}(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu}$$

According to the design specification mentioned in the above section used the gain, phase and phase derivative of the FOPID controller

The gain, phase and phase derivative of FOPID controller is given by the following equations

The phase of Fractional Order PID controller is given by

$$Arg\left[C(\mathrm{j}\omega)\right] = \tan^{-1}\left(\frac{K_d\omega^{\mu}\sin\frac{\mu\pi}{2} - K_i\omega^{-\lambda}\cos\frac{\lambda\pi}{2}}{K_p + K_i\omega^{-\lambda}\cos\frac{\lambda\pi}{2} + K_d\omega^{\mu}\cos\frac{\mu\pi}{2}}\right) \tag{6.7}$$

Gain of Fractional Order PID controller is given by

$$\left|C(\mathrm{j}\omega)\right| = \sqrt{(K_p + K_i\omega^{-\lambda}\cos\frac{\lambda\pi}{2} + K_d\omega^{\mu}\cos\frac{\mu\pi}{2})^2 + (K_d\omega^{\mu}\sin\frac{\mu\pi}{2} - K_i\omega^{-\lambda}\cos\frac{\lambda\pi}{2})^2} \tag{6.8}$$

The derivative of phase of the Fractional Order PID controller w.r.t. ω

$$\left(\frac{d}{d\omega}\left(Arg[C(\mathrm{j}\omega)]\right)\right)=\frac{\left(K_pK_d\mu\omega^{\mu-1}\sin\frac{\mu\pi}{2}\right)+\left(K_pK_i\lambda\omega^{-\lambda-1}\sin\frac{\lambda\pi}{2}\right)+\left(K_dK_i(\lambda+\mu)\omega^{-\lambda+\mu-1}\sin\frac{(\lambda+\mu)\pi}{2}\right)}{\left(K_p+K_i\omega^{-\lambda}\cos\frac{\lambda\pi}{2}+K_d\omega^{\mu}\cos\frac{\mu\pi}{2}\right)^2+\left(K_d\omega^{\mu}\sin\frac{\mu\pi}{2}-K_i\omega^{-\lambda}\cos\frac{\lambda\pi}{2}\right)^2} \tag{6.9}$$

6.2.1 NON-LINEAR MINIMIZATION PROBLEM

According to the specifications mentioned in the previous section a set of five non linear Equations (6.2) to Equation (6.6) in terms of five unknown parameters (K_p, K_i, K_d, λ, μ) is found. The complexity of this set of non linear equations is very significant, especially when it is used a $PI^{\lambda}D^{\mu}$ controller and fractional order of Laplace variable's' are introduced. Therefore, it is difficult to solve these five non linear equations in a easy and direct way. An optimization tool box in MATLAB is used to solve these non linear equations with minimum error. Here we are employing FMINCON function that calculates the non-linear constraints minimum of a function of several valuables.

$$\min_{x}\ f(x)\,\text{such that}\begin{cases} c(x)\le 0 \\ ceq(x)=0 \\ A.x\le b \\ Aeq.x=beq \\ Lb\le x\le ub \end{cases}$$

It solves the problems of the form MIN $f(x)$ subjected to $C(x)\le 0$, $Ceq=0$, $LB\le x\le UB$, where f is the function to minimize; C and Ceq represent the non linear constraints and linear constraints. X

is the minimum looked for *LB* and *UB* define a set of lower and upper bounds on the design variables, x.

Here the robustness specification Equation (6.4) is considered as the objective function to minimize and other specifications are taken as non linear constraints for the minimization and all of them subjected to the optimization parameters defined within the function FMINCON.

6.3 ILLUSTRATIVE EXAMPLES-I

Here we considered the position servo system and AC servo system for which we are designing the fractional order PID controllers. To demonstrate the superiority of the proposed fractional order PID controller a conventional PID controller is designed for the same plant and compared the responses.

POSITION SERVO SYSTEM

Consider the negetive unity feedback position servo system given by the transfer function

$$P(s) = \frac{0.55}{62s + 1} \tag{6.10}$$

Design specifications for the controlled system has been follows

- Gain crossover frequency, ω_c=1 rad/sec
- Phase margin, φ_m= 0.44π ≈ 80 deg.
- Robustness to variations in the gain of the plant must be fulfilled
- High frequency noise rejection(Complementary Sensitivity Function

$$\left|T(j\omega)\right|_{dB} \le -20dB, \forall \omega \ge \omega_t = 10rad/\sec$$

- Good output disturbance rejection rejection(Sensitivity Function)

$$\left|S(j\omega)\right|_{dB} \le -20dB, \forall \omega \le \omega_s = 0.01rad/\sec$$

Now, we will design the FOPID controller by position servo plant, the steps given below

I. Given, the gain crossover frequency, phase margin, robustness specification, sensitivity and complementary sensitivity functions

II. With these specifications we write Equation (6.2) to Equation (6.6), it will give us five non linear equations in terms of K_p, K_i, K_d, λ, μ.

III. Determine the values of these five parameters by FMINCON optimization tool box in MATLAB. The values comes out to be, $K_p=110.32$, $K_i=9.7086$, $K_d=0.559$, $\lambda=0.515$, $\mu=0.4109$.

IV. With the obtained values of K_p, K_i, K_d, λ, μ obtain the transfer function of the FOPID controller.

The fractional order PID controller comes out to be

$$C_{FOPID}(s) = 110.32 + \frac{9.7086}{s^{0.515}} + 0.559 s^{0.41092} \tag{6.11}$$

V. Approximate the FOPID controller to integer order by using oustapp command in fomcon toolbox of MATLAB (take frequency band 0.0001 to 1000000).

6.3.1 RESULTS

6.3.1.1 Step response of the closed loop system

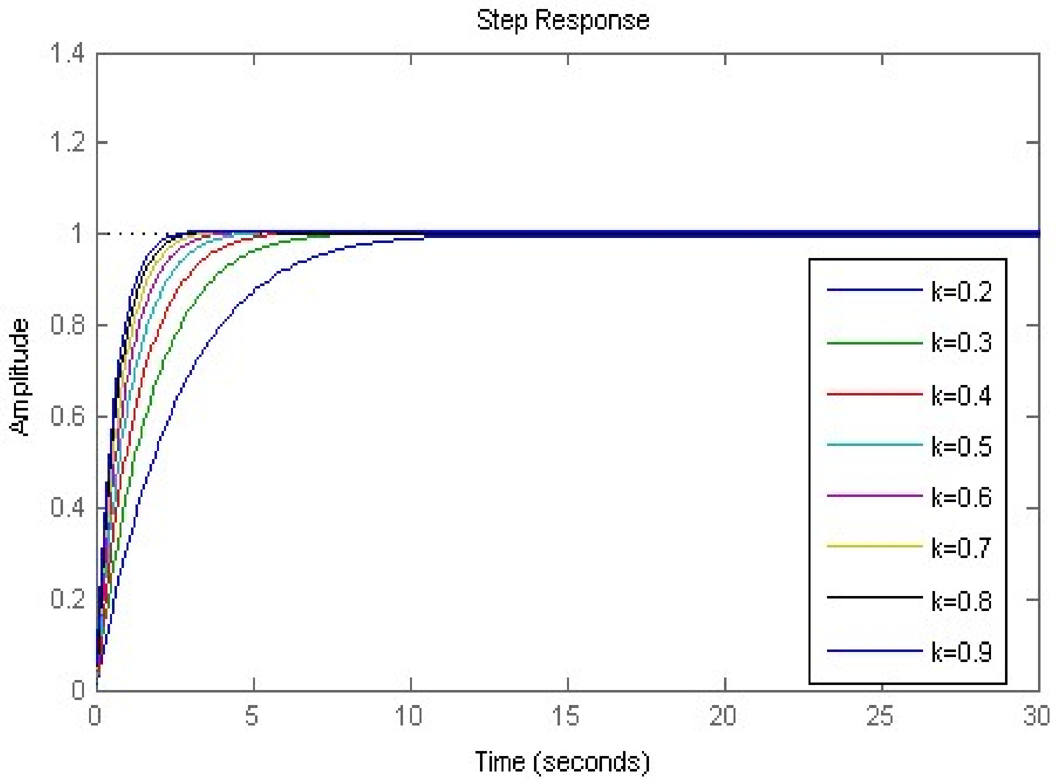

Figure 6.1- Step response of the closed loop system for 0.2≤k≤0.9

Figure 6.1 represents the step response of the closed loop system. From Figure 6.1 we can observe that the overshoot of the closed loop system is constant for gain variation. It proves that the control system obeys the iso-damping properity.

6.3.1.2 Open loop Frequency response

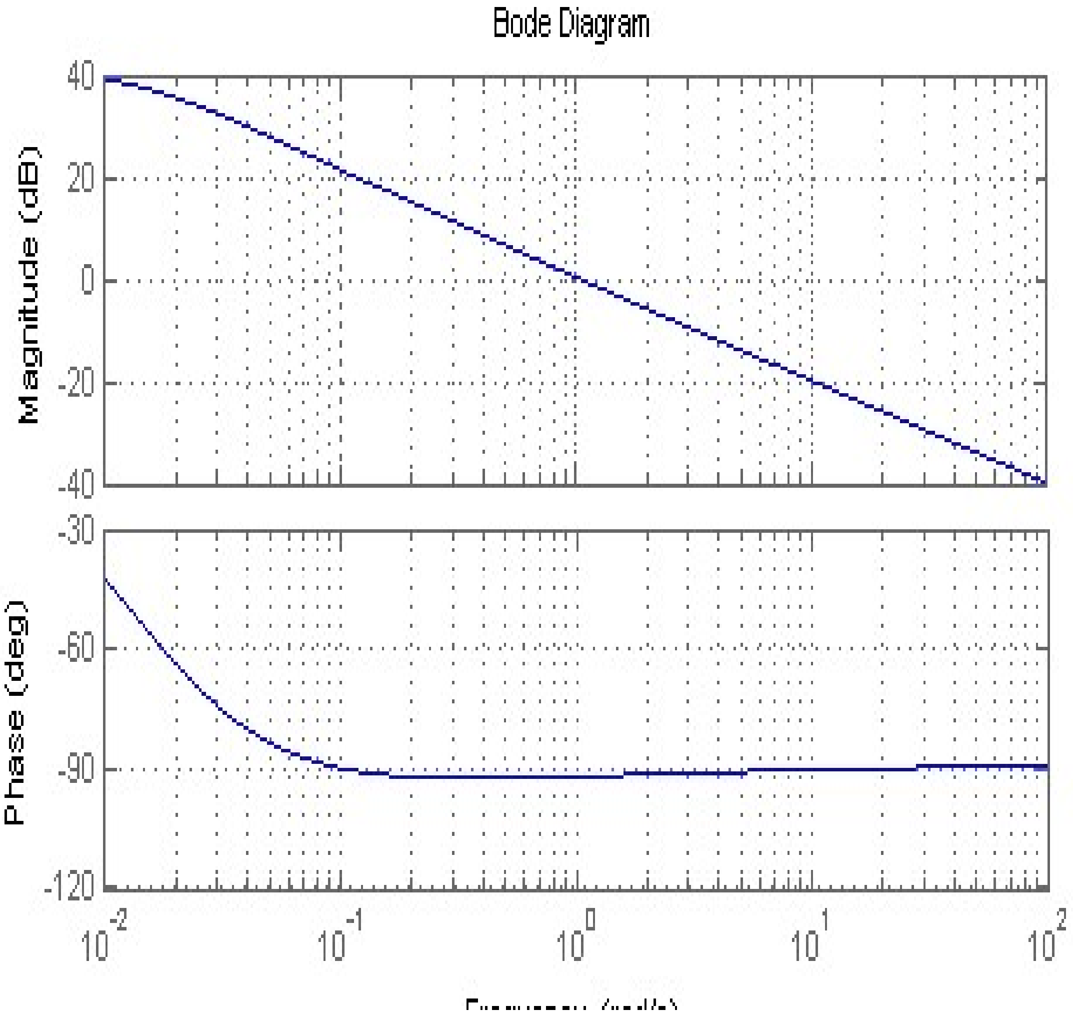

Figure 6.2- Bode plot of the open loop system

The open loop frequency response of the system is shown in Figure 6.2. From the Figure 6.2 we can observe that the user design specification such as gain cross over frequency(ω_{gc}=1 rad/sec) and phase margin($\varphi_m=80^0$) are achieved. also the open loop phase of the system is flat around the gain crossover frequency this means the third specification of robustness is achieved.

6.3.1.3 Sensitivity Function

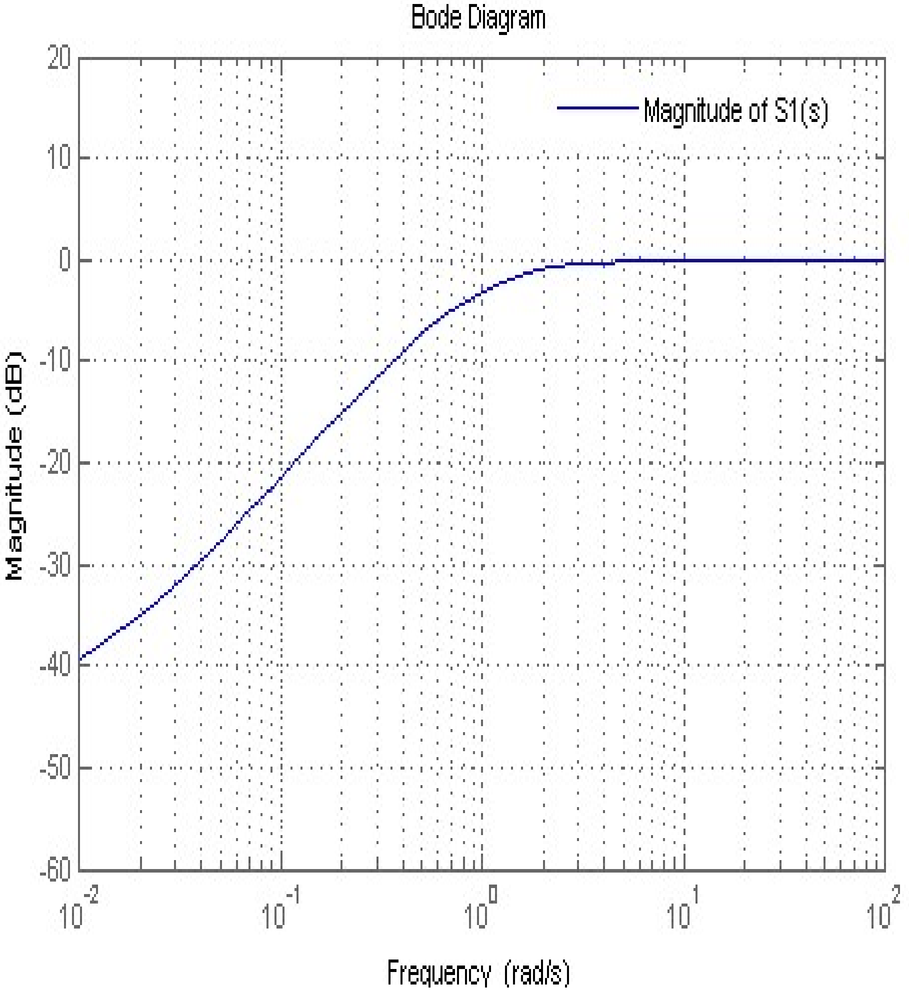

Figure 6.3- Magnitude of S(s)

The Sensitivity Function
Specification is fulfilled
$\forall\omega \leq \omega_s = 0.01 \text{rad} / \text{sec}$

The magnitude plot of the sensitivity function S(s) is shown in Figure 6.3. As it can be observed that $|S(j\omega)| \leq -20\text{dB}$ for frequencies $\omega \leq \omega_s = 0.01$rad/sec i.e., the output disturbance is -20dB for all frequencies less than 0.01 rad/sec. The fourth specification of sensitivity function is fulfilled.

6.3.1.4 Complementary Sensitivity Function

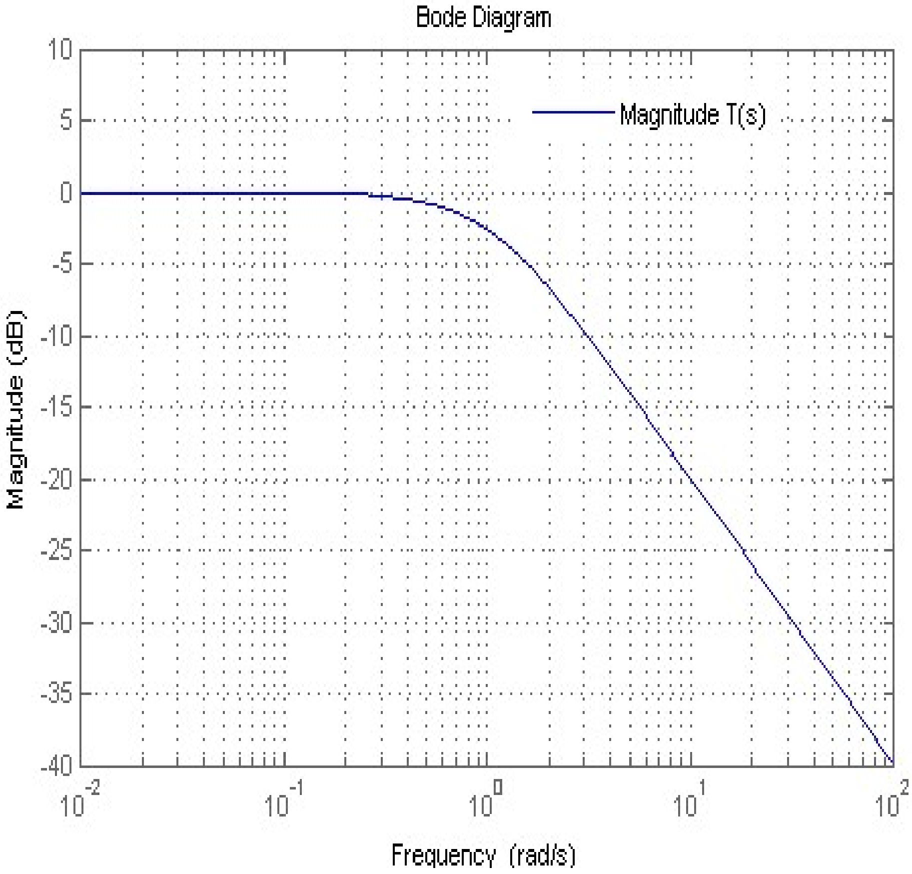

Figure 6.4- magnitude of T(s)

The Complementary Sensitivity
Function Specification is fulfilled

$$\forall \omega \geq \omega_s = 10 \mathrm{rad/sec}$$

The magnitude plot of the complementary sensitivity function T(s) is shown in Figure 6.4. It can be observed that $|Tj\omega)| \leq -20$dB for frequencies $\omega \geq \omega_t = 10$ rad/sec i.e., the unwanted noise is -20dB for all frequencies greater than 10 rad/sec. The fifth specification of complementary sensitivity function is achieved.

For the fair comparison let design the integer order PID controller the same plant. By employing the classical Ziegler Nicholas Technique for tuning of Integer Order PID controller and the resultant controller is

$$C_{IOPID}(s) = 16.4 + \frac{1.1515}{s} + 0.01s \qquad (6.12)$$

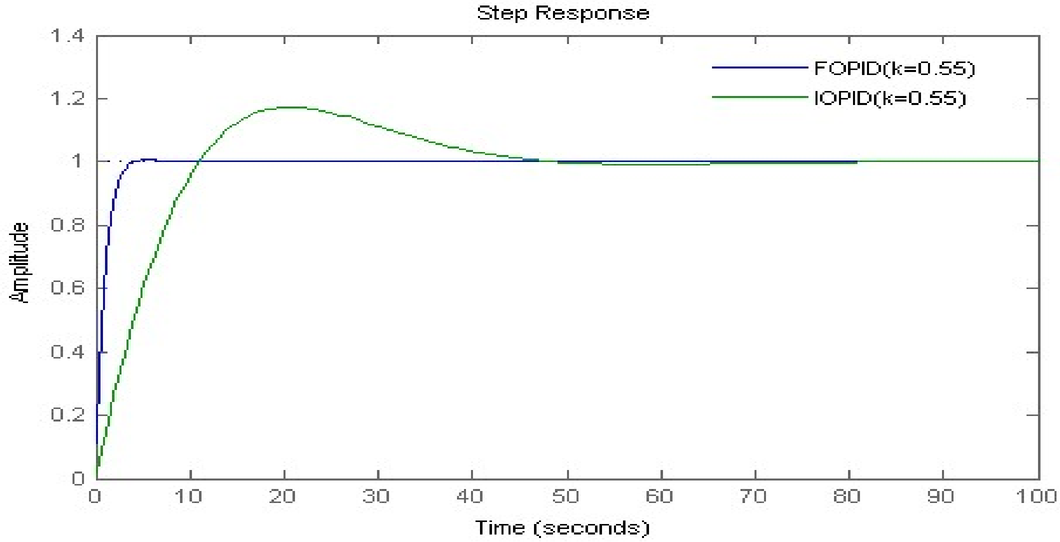

Figure 6.5- step response of FOPID and IOPID for k=0.55

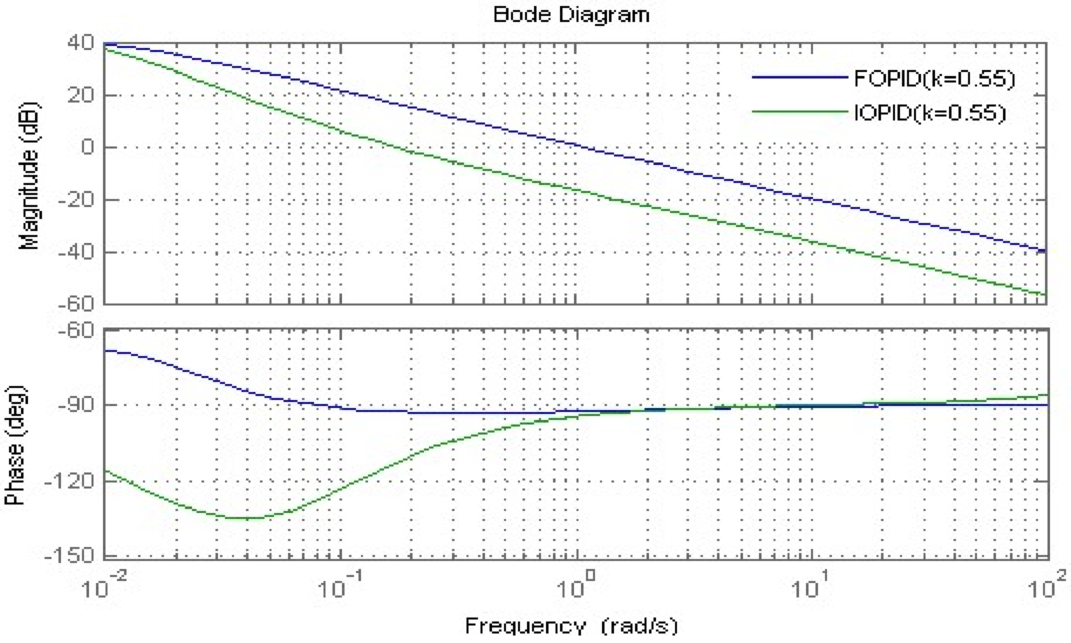

Figure 6.6- Bode plot of FOPID and IOPID for k=0.55

Compensation type	**Values of controller parameters**	**Peak over shoot**	**Settling time**	**ISE**	**IAE**
FOPID compensation (k=0.55)	K_p=15.28,K_i =98.126 K_d=1.1625, μ=1.0148. λ=1.1625	1.003	3.01sec	0.4763	0.9245
IOPID Compensation (k=0.55)	K_p=16.4, K_i=-1.515., K_d =0.012	1.17	42.6sec	2.831	4.535

Table 6.1 Performance of the of FOPID & IOPID controllers

It is evident from the above Table 6.1 that the proposed Fractional Order PID controller outperforms the integer order PID in terms of peak over shoot, settling time and rise time. Peak overshoot betters from 1.17 to 1.003 and settling time is fast in case of fractional order PID controller. The control system performance index measured by means of Integral square error (ISE) and Integral absolute error (IAE). By employing FOPID controller for this system ISE reduces from 2.831 to 0.4763 and IAE reduces from 4.535 to 0.9245.

6.4 ILLUSTRATIVE EXAMPLE-II

Consider the negetive unity feedback AC servo system given by transfer function

$$P(s)=\frac{0.4}{s(2.7736s+1)} \tag{6.13}$$

Design specifications for the controlled system has been follows

- Gain crossover frequency, ω_c=1 rad/sec
- Phase margin, φ_m= 50 deg.
- Robustness to variations in the gain of the plant must be fulfilled
- High frequency noise rejection(Complementary Sensitivity Function

$$\left|T(j\omega)\right|_{dB} \leq -20dB, \forall \omega \geq \omega_t = 10rad/\sec$$

- Good output disturbance rejection rejection(Sensitivity Function)

$$\left|S(j\omega)\right|_{dB} \leq -20dB, \forall \omega \leq \omega_s = 0.01rad/\sec$$

Now, we will design the FOPID controller by AC servo plant, the steps given below

I. Given, the gain crossover frequency, phase margin, robustness specification, sensitivity and complementary sensitivity functions

II. With these specifications we write Equation (6.2) to Equation (6.6), it will give us five non linear equations in terms of K_p, K_i, K_d, λ, μ.

III. Determine the values of these five parameters by FMINCON optimization tool box in MATLAB. The values comes out to be, K_p=0.19927, K_i=1.0688, K_d= 16.661, λ=0.47377 , μ=0.50223 .

IV. With the obtained values of K_p, K_i, K_d, λ, μ obtain the transfer function of the FOPID controller.

The fractional order PID controller comes out to be

$$C(s) = 0.19927 + \frac{1.0688}{s^{0.47377}} + 16.661 s^{0.50223} \tag{6.14}$$

V. Approximate the FOPID controller to integer order by using oustapp command in fomcon toolbox of MATLAB (take frequency band 0.0001 to 1000000).

6.4.1 RESULTS

6.4.1.1 Step response of the closed loop system

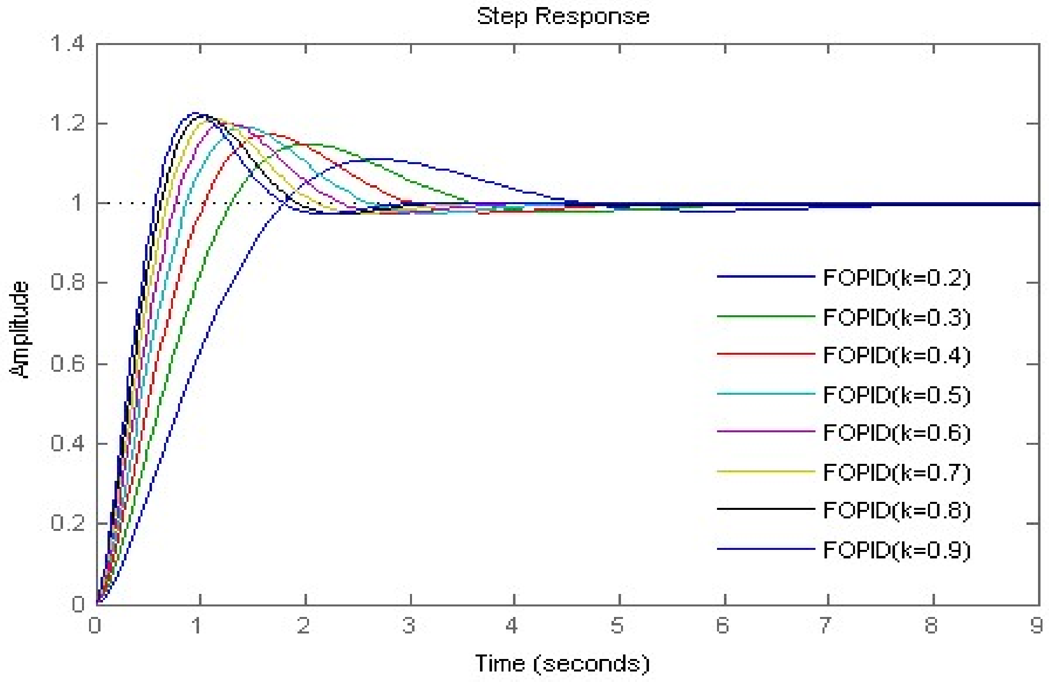

Figure 6.7- Step response of the closed loop system for 0.2≤k≤0.9

Figure 6.7 is the step response of the closed loop system. From Figure 6.7 we can observe that the overshoot of the closed loop system is constant for gain variation. It proves that the control system obeys the iso-damping properity.

6.4.1.2 Open loop Frequency response

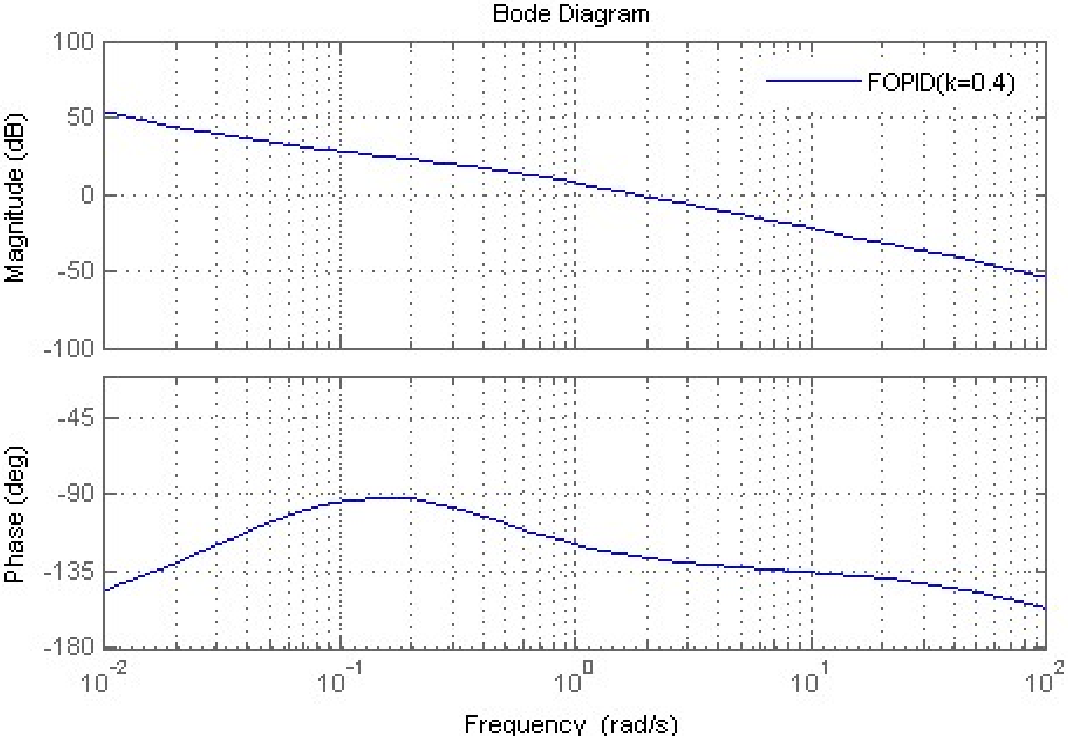

Figure 6.8- Bode plot of the open loop system

The open loop frequency response of the system is shown in Figure 6.8. From the Figure 6.8 we can observe that the user design specification such as gain cross over frequency (ω_{gc}=1 rad/sec) and phase margin ($\varphi_m=50^0$) are achieved. Also the open loop phase of the system is flat around the gain crossover frequency this means the third specification of robustness is achieved.

6.4.1.3 Sensitivity Function

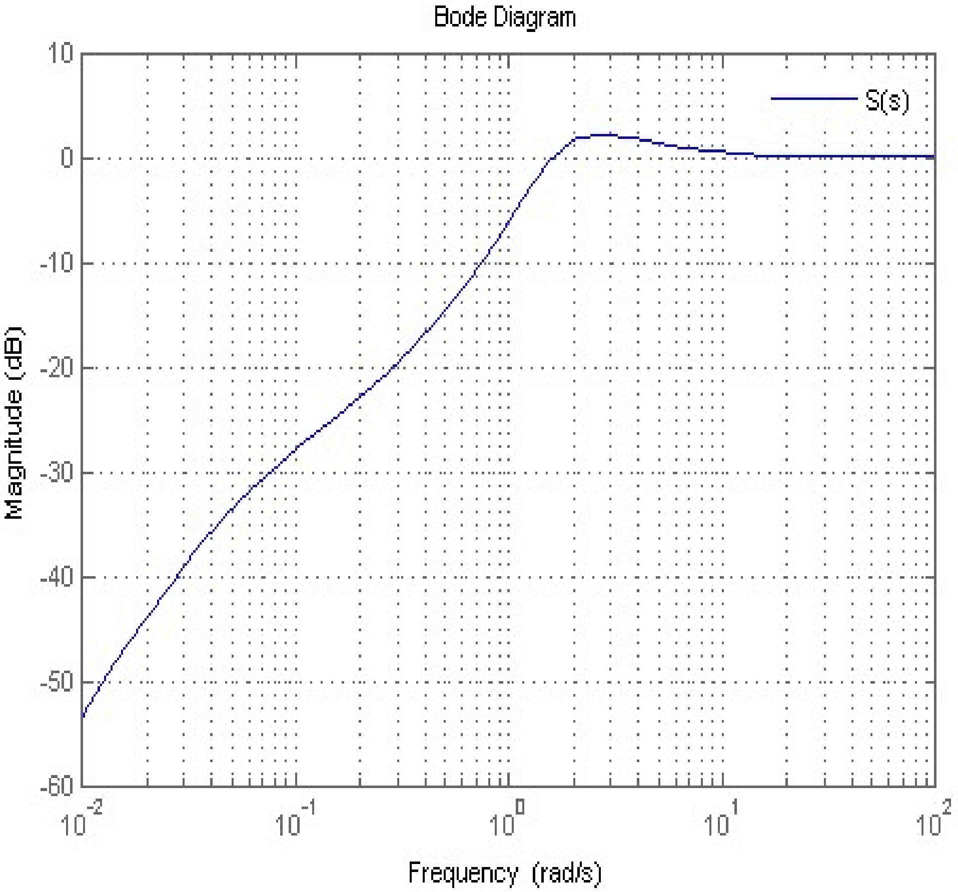

Figure 6.9-Magnitude of S(s)

The Sensitivity Function
Specification is fulfilled
$\forall \omega \leq \omega_s = 0.01 \text{rad} / \text{sec}$

The magnitude plot of the sensitivity function S(s) is shown in Figure 6.9. As it can be observed that $|S(j\omega)| \leq -20\text{dB}$ for frequencies $\omega \leq \omega_s = 0.01\text{rad/sec}$ i.e., the output disturbance is -20dB for all frequencies less than 0.01 rad/sec. The fourth specification of sensitivity function is fulfilled.

6.4.1.4 Complementary Sensitivity Function

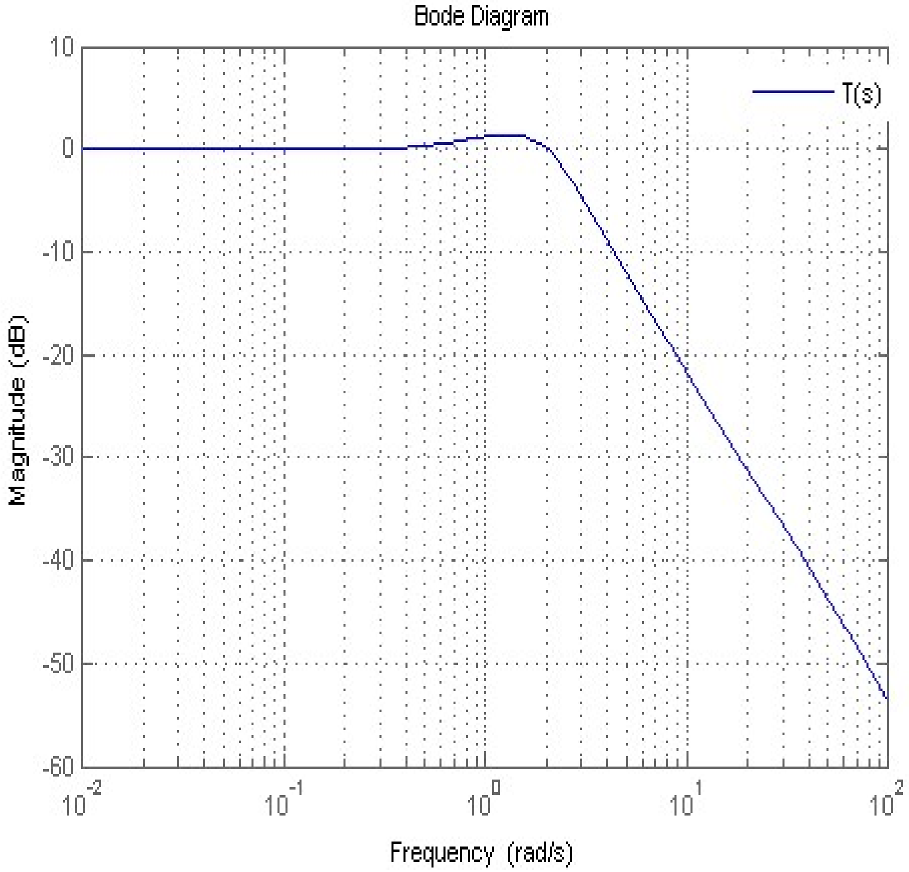

Figure 6.10- magnitude of T(s)

The Complementary Sensitivity
Function Specification is fulfilled

$$\forall\omega \geq \omega_s = 10\text{rad} / \sec$$

The magnitude plot of the complementary sensitivity function T(s) is shown in Figure 6.10. It can be observed that |Tjω)| ≤-20dB for frequencies $\omega \geq \omega_t$=10 rad/sec i.e., the unwanted noise is -20dB for all frequencies greater than 10 rad/sec. The fifth specification of complementary sensitivity function is achieved.

For the fair comparison let design the integer order PID controller the same plant. By employing the classical Ziegler Nicholas Technique for tuning of Integer Order PID controller and the resultant controller is

$$C_{IOPID}(s) = 1.08 + \frac{0.1946}{s} + 1.525s \tag{6.15}$$

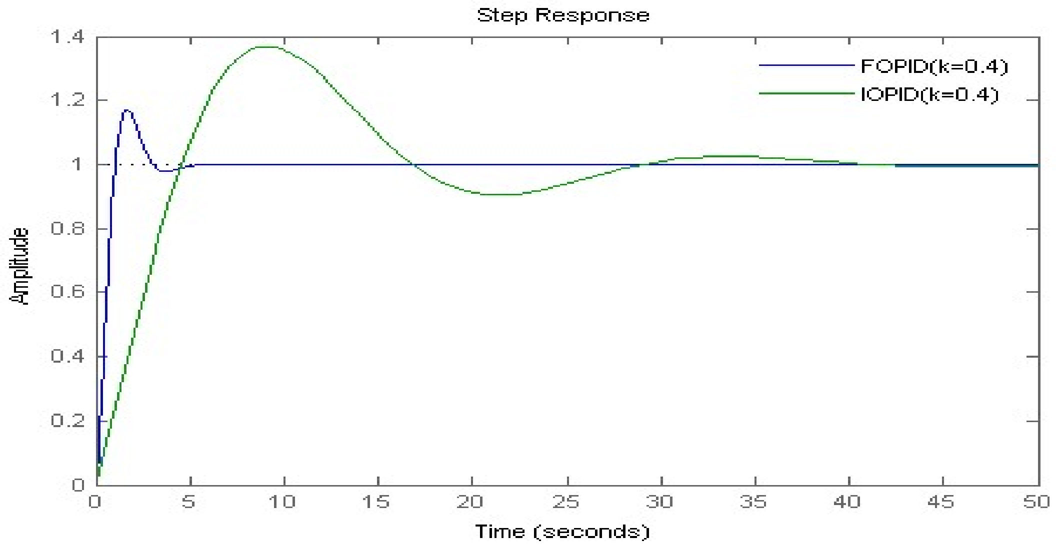

Figure 6.11- step response of FOPID an IOPID

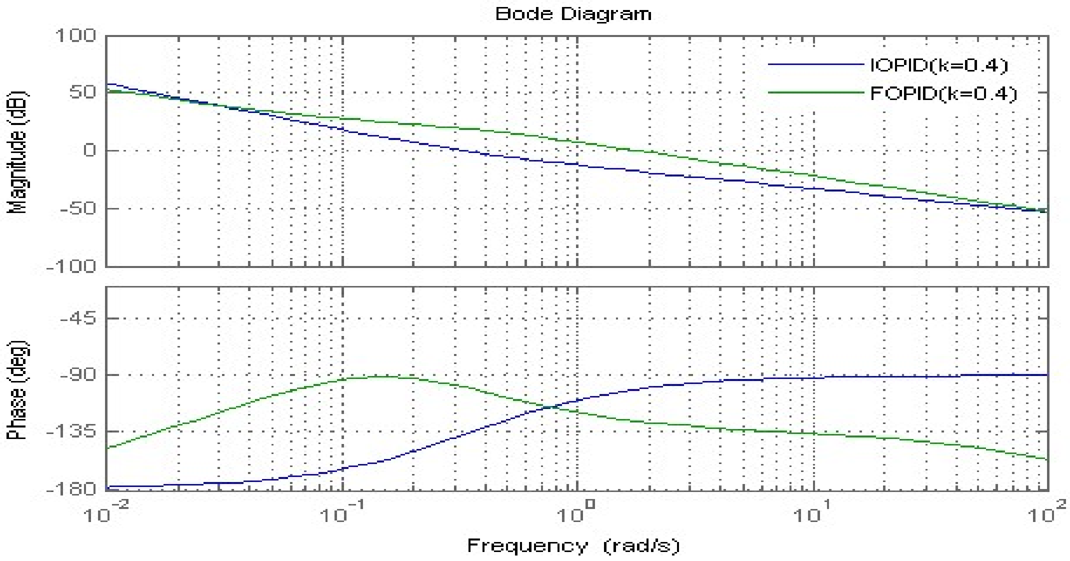

Figure 6.12- Bode plot of FOPID and IOPID

Compensation type	**Values of controller parameters**	**Peak over shoot**	**Settling time**	**ISE**	**IAE**
FOPID compensation (k=0.40)	K_p=0.19927, K_d=16.661, K_i=1.0688 μ=0.5022, λ=0.4737	1.17	4.06sec	0.3824	0.7425
IOPID compensation (k=0.40)	K_p=1.089, K_d=1.525. K_i=0.1946	1.37	36.1sec	1.763	3.276

Table 6.2- performance Comparison of FOPID controller and IOPID controller

It is evident from the above Table 6.2 that the proposed Fractional Order PID controller outperforms the integer order PID in terms of peak over shoot, settling time and rise time. Peak overshoot betters from 1.17 to 1.003 and settling time is fast in case of fractional order PID controller. The control system performance index measured by means of Integral square error (ISE) and Integral absolute error (IAE). By employing FOPID controller for this system ISE reduces from 2.831 to 0.4763 and IAE reduces from 4.535 to 0.9245.

6.5 CONCLUSION

In this chapter, an analytical method is applied for design of FOPID in frequency domain. The designed controller achieves five different specifications for the two plants. An optimization toolbox in MATLAB called FMINCON is employed to design the FOPID controller. With the help of simulation results, we can prove that the design specifications are fulfilled precisely. Thus taking the advantage of λ (integration action) and μ (derivative action) to fulfill additional specifications. The designed controller guarantees the closed loop control system is robust for gain changes and external disturbances. The proposed FOPID controller outperforms the classical PID controller in terms of time domain specifications as well as frequency domain specifications.