PREFACE

Gas dynamics, a branch of fluid dynamics, focuses on the behaviour of gases in motion. The formulation of the conservation laws of mass, momentum, and energy in the 19th and 20th centuries established a theoretical framework that facilitated the analysis of compressible and incompressible flows. Since high-speed flying became possible, gas dynamics has grown into a field of study that includes physics, chemistry, applied mathematics, and astrophysics. Although the name implies gaseous states, the field consists of assessments of pressure, temperature, density, and velocity fluctuations under various flow circumstances.

A wave can be viewed as a propagating disturbance, any identifiable feature that travels through or between media at a considerable speed. This feature might form as crests and troughs or as a sudden change in some physical quantity, as long as one can detect it and track its position over time. Although a wave's shape may distort, its amplitude may vary, or its velocity may shift, it remains recognizable as long as its defining characteristic persists. Several wave shapes can be accurately analyzed mathematically using hyperbolic partial differential equations.

The Riemann problem is a cornerstone in the theory of hyperbolic partial differential equations, especially within gas dynamics and fluid mechanics. First posed by Bernhard Riemann in 1860, it considers a one-dimensional gas flow that begins with two distinct constant states separated by a discontinuity. This simple setup models how shock waves and rarefaction waves form and propagate.

In gas dynamics, a shock wave is a moving surface across which flow variables (pressure, density, and velocity) undergo an abrupt jump. In contrast, a rarefaction wave is a continuous expansion fan where these quantities vary smoothly. A contact wave allows velocity and pressure to remain continuous while density, temperature, and entropy change discontinuously.

Since nonlinear partial differential equations typically fail to admit globally smooth solutions, even initial continuous data can develop discontinuities (shocks) in finite time. The Riemann problem captures this fundamental behaviour and provides exact solutions to specific nonlinear systems, making it an invaluable tool for understanding and approximating more complex flows.

Simple waves are fundamental in studying hyperbolic PDEs and nonlinear wave propagation and are crucial for understanding more complex wave phenomena. This thesis aims to generate several canonical models of nonlinear waves in hyperbolic systems. While designing and solving these models, an attempt is made to find analytical solutions with which one can examine the behaviour of nonlinear waves in different regimes of gas dynamics and magnetohydrodynamics systems. The existence of simple waves, solving Riemann problems, and the diffraction-reflection phenomenon of weak shock by rigid wedges have been studied.

In the present thesis, five problems have been considered. For these five problems, many results for quasilinear hyperbolic systems have been obtained using analytical solution methods applicable to specific regimes of hyperbolic PDEs. The thesis has been designed so that existing methods, such as the characteristic decomposition method, differential constraint method, and asymptotic expansions, can be used to solve the considered models. The whole thesis is divided into six main chapters as follows:

Chapter 1 defines the key terminology used throughout the thesis and provides a concise overview of the underlying mathematical theory and its fundamental properties. The physical characteristics of hyperbolic systems, equations of state (non-ideal gas, generalized Chaplygin gas, and a more realistic extended Chaplygin gas), and the primary methods employed in this work are also briefly reviewed. Lastly, several crucial findings that are required for subsequent chapters are provided.

In **Chapter 2**, a method called characteristic decomposition is used to show the presence of simple waves for the two-dimensional compressible flow in a non-ideal magnetohydrodynamics system. In this chapter, a steady and pseudo-steady state magnetohydrodynamics system is considered, and we provide a characteristic decomposition of the flow equations in both systems. This decomposition ensures the presence of a simple wave adjacent to a region of constant state for the system under consideration. Further, this result is extended as an application of the characteristic decomposition in a pseudo-steady state, and we prove the existence of a simple wave in a full magnetohydrodynamics system by taking the vorticity and the entropy to be constant along the pseudo-flow characteristics. These results extend the fundamental theorem proposed by Courant and Friedrichs for a reducible system.

Chapter 3 presents significant findings on reducible equations in Courant and Friedrichs's seminal work. In this chapter, we discuss the presence of simple waves in a 2D magnetohydrodynamic system with an anti-van der Waals-modified Chaplygin gas. Using a sufficient condition for the characteristic decomposition of a strictly hyperbolic system, we establish the existence of simple waves for a non-reducible system and extend the fundamental finding of Courant and Friedrichs, which was originally proposed for a reducible system.

Chapter 4 obtains the solution to the non-homogeneous Riemann problem by applying the approach of differential constraint for the one-dimensional ($1D$) generalized Chaplygin gas equations with non-constant initial data. Here, we take the source term as a Coulomb-type constant frictional term. We reduced the governing non-homogeneous model into a homogeneous one by introducing a new velocity variable. By virtue of this advantage of the frictional-type source term, we would easily determine the solution of a reformulated homogeneous governing model. We have computed the specified model's differential constraint and its consistency conditions. Moreover, we have derived the compatibility condition between the governing model and the differential constraints. The solutions to the generalized Riemann problem for the $1D$ Euler's equation of the governing gas model are obtained, as well as for the smooth and non-constant initial conditions. A comprehensive overview of the solutions is studied.

Chapter 5 analyzed the diffraction and reflection phenomena of a weak shock interacting with a right-angled wedge in the context of a more realistic extended Chaplygin gas. Asymptotic solutions to the two-dimensional Euler system have been derived under appropriate boundary conditions that characterize the diffraction of a weak shock from the wedge. In this work, the effects of the specific gas considered are carefully modeled, and their influence on the overall flow configuration is examined. In particular, a detailed investigation of the local structure near a singular point is conducted, highlighting the significant role of the considered gas behaviour in shaping the flow dynamics.

Chapter 6 investigates the diffraction and reflection phenomena resulting from the interaction of a weak shock wave with a wedge at an arbitrary angle, utilizing a more realistic extended Chaplygin gas model. Asymptotic solutions to the two-dimensional Euler equations have been systematically derived, incorporating boundary conditions that accurately describe the diffraction of a weak shock wave encountering the wedge. The work rigorously accounts for the specific properties of the gas under consideration and evaluates their impact on the overall flow structure. A particular focus is placed on the detailed analysis of the local flow characteristics near a singular point, emphasizing the critical influence of the gas's unique behaviour on the resulting flow dynamics.

In **Chapter 7**, the key findings of this thesis are synthesized, and significant contributions to the analytical understanding of nonlinear waves, generalized Riemann problems, and shock reflection-diffraction phenomena within realistic gas models are highlighted. Moreover, this chapter outlines promising avenues for future research, including the exploration of multiphase and three-dimensional flows, global solutions for transonic configurations, and numerical validation of analytical predictions. These suggested future directions emphasize extending current analytical methods to broader contexts, potentially bridging the gap between rigorous mathematical analysis and real-world physical applications.