Chapter 1

Introduction

Optimization is a method used to identify the most suitable option from a set of alternatives that meet specific criteria, aiming to utilize available resources efficiently. As a result, optimization finds broad application across various domains, such as navigation and guidance of aerial and ground autonomous vehicles, machine learning, economics, computational systems biology, and more.

Continuous-time optimization is presently a highly active area of research within optimization theory [5]. Continuous-time optimization involves designing a dynamical system with a stable equilibrium point that corresponds to the optimal solution of the optimization problem. In contrast to discrete-time algorithms, analysis of continuous models is not plagued by step size selection in related iterative methods. Their continuous-time counterparts, obtained by considering infinitesimal step sizes, are described by differential equations. The theory of ordinary differential equations provides valuable insights and tools for understanding and developing optimization methods. The continuous-time formulation of optimization problems enables simple and elegant convergence proofs to equilibrium points, leveraging control theory tools such as Lyapunov stability theory.

First-order iterative algorithms are widely used in machine learning to handle large-scale datasets and leverage parallel processing architectures. Gradient-based algorithms form a class of optimization methods known for their robustness to measurement errors, ease of numerical implementation, and low memory requirements [1]. A classic example is the gradient flow (GF), defined by $\dot{x} = -\nabla g(x)$, where $g : \mathbb{R}^n \to \mathbb{R}$ is a continuously differentiable, unconstrained objective function that we aim to minimize. It is well known that the strict minima of a locally convex function $g : \mathbb{R}^n \to \mathbb{R}$ are stable equilibria of the

GF dynamics, and that, if the sublevel sets of g is compact, then the trajectories converge asymptotically to the set of critical points of g.

Classical techniques include dual methods, Lagrangian multiplier techniques, penalty (barrier) function approaches, and projected gradient techniques to solve constrained optimization problems. The optimization problems, including inequality constraints, are handled by extending first-order conditions to Karush-Kuhn-Tucker (KKT) conditions.

The approaches discussed above for solving optimization problems generally assume a static setting, where neither the objective function nor the constraints change over time. However, in many real-time applications, optimization problems are dynamic—meaning the objective function or constraints evolve based on certain parameters. In time-varying optimization (TVO) problems, either the objective or the constraints are explicitly time-dependent and change continuously with time, leading to an optimizer trajectory composed of the optimal solutions at each moment.

A traditional approach to solving time-varying optimization (TVO) problems involves sampling the objective and constraint functions at specific time instants and solving the resulting optimization problem at each of these points. This method demands significant computational effort and ensures optimality only if each sampled problem converges within a time interval shorter than the sampling period. An alternate useful approach is based on time-structured algorithms. Time-structured algorithms utilize the characteristics of an optimization problem, such as differentiability, Lipschitzness, etc. A widely used time-structured approach is the prediction-correction method, which consists of a "prediction" step to estimate the optimizer trajectory at the next time instant, followed by a "correction" step to adjust the prediction toward the actual optimizer path. However, this algorithm typically relies on the strong convexity of the objective function to effectively handle both constrained and unconstrained time-varying optimization problems.

The perspective of continuous-time optimization not only provides valuable insights and concepts for analyzing and developing algorithms but also offers intuitive methods for designing control strategies for physical systems. Gradient flows can be employed to analyze system structure and properties directly from data, and this insight can then be leveraged for controller design. Notably, key properties such as dissipation inequalities, L_2-gain bounds, and passivity enable the use of established feedback theorems to develop controllers with guaranteed performance. To intuitively grasp the concepts of dissipativity and passivity, one can imagine H as a physical system whose energy increases only when supplied by an external source. A simple example is an RLC circuit connected to an external battery. In control design based on passivity properties, knowing the plant's passivity measures is useful. Specifically, quantitative measures of passivity, such as output feedback passivity (OFP), indicating a shortage of passivity in a plant. Typically, a mathematical model is needed to determine the passivity index, but passivity indices can be formulated as an optimal solution to an optimization problem. Hence, gradient flows are used to calculate passivity indices using a data-driven approach.

The passivity indices of a system are useful when designing a controller for its stabilization. One of the main results for passive systems states that the negative feedback interconnection of two passive systems is passive. When a system is not passive or has a shortage of passivity, we can interconnect it with a system with excess passivity to make the closed-loop system passive. Passivity and Lyapunov stability are related; we can make constructive use of this relationship. Conventional stability concepts for dynamical systems with locally Lipschitz or Lipschitz dynamics have been extensively studied in the literature. Unlike asymptotic stability or exponential stability, which describe the convergence of system trajectories as time approaches to infinity, finite-time stability (FTS) ensures that convergence occurs within a finite time interval. This pursuit not only enhances the convergence speed but also contributes to a more precise and efficient control scheme, making significant area of interest in advancing control system theories and applications.

1.1 Motivation

Gradient flow is one of the time-honored and famous results of continuous-time optimization. The standard gradient flow system is asymptotically or exponentially convergent, meaning that a strict optimal solution is obtained when the time approaches infinity. Asymptotic/exponential convergence of the solution to the optimal point is not desirable for time-critical tasks or multi-cycle optimization.

It is well known that since the introduction of finite-time stability, the system and control community has put a great deal of effort into obtaining less conservative convergence, resulting in many new concepts and results such as finite-time/fixed-time stability.

Then, a problem naturally arises: is it possible to design more efficient gradient flow systems with the help of more advanced stability theory? Thus, the gradient dynamical systems ensuring finite-time or fixed-time convergence have received a lot of attention. Normalized and signed gradient flows guarantee finite-time convergence, where the upper bound of settling time is related to the initial gradient value. Fixed-time gradient flows guarantee fixed-time convergence, where the upper bound of settling time is uniform irrespective of the initial conditions. The upper bound of the settling time is often difficult to specify, either depending on system initialization or involving complex parameter calculation. One of the motivations of this thesis is to provide an answer to the following question: how could one modify the gradient flow dynamics so that the trajectories converge to the critical points of the function in a prior chosen time?

The main motivation of this thesis is to explore the interplay between optimization and control theory. Control theory is rich and fruitful, resulting in a plethora of mathematical tools. In this direction, control theory can be used to guarantee the convergence of algorithms to accurate solutions and to analyze the impact of numerical errors and computational delays. Lyapunov stability theory provides elegant and simple proofs for the convergence of solutions to optimal points. Control-theoretic tools have been widely applied in the context of time-invariant optimization problems. This thesis uses the tools of control theory to develop algorithms to solve TVO problems with equality and inequality constraints. Further, advanced studies related to Lyapunov stability theory are used in convergence analysis. This thesis also investigates the applicability of the developed novel approaches to solve TVO problems in a real-world example like the navigation of a robot in an environment with circular obstacles.

Furthermore, the continuous-time optimization perspective open doors for designing controllers without explicit knowledge of the mathematical model of the system. This thesis delves in analyzing the structure and system properties from input-output data from simulations or experiments. Due to its relevance in controller design, we consider the problem of determining whether and, if so to which extent is an input-output system is passive. In control design based on passivity properties, knowing the plant's passivity measures is useful. Specifically, quantitative measures of passivity, such as output feedback passivity (OFP) and input feedforward passivity (IFP) indices, offer suitable methods for controller design. To determine IFP and OFP indices, their definitions can be reformulated as an optimization problem. To determine optimal input-output samples corresponding to IFP and OFP properties, gradient flows can be employed. Accelerated gradient techniques are important in the case of data-driven optimization problems. This thesis explores a modified gradient flow, with a predefined-time convergence property to find passivity indices.

As one of the main results for passive systems states that the negative feedback interconnection of two passive systems is passive. Building upon this idea, the motivation is to utilize the calculated passivity indices and feedback interconnection property of passive systems to stabilize the origin of the closed-loop system in finite time without the knowledge of the mathematical model of the system.

1.2 Literature Review

Continuous-time optimization is currently a vibrant area of research within optimization theory, with prior studies offering valuable insights and elegant techniques for establishing stability and convergence properties. This literature review offers a comprehensive overview of current research in the field of continuous-time optimization, encompassing various gradient flows, unconstrained and constrained time-varying optimization problem-solving techniques, analysis of system properties using gradient flows, and related areas. It aims to illuminate the key contributions and emerging trends within this dynamic field.

1.2.1 Gradient Flows

Guidelines on how to design dynamical systems for optimization purposes, with a special emphasis on gradient systems, are described in [6]. Continuous-time negative gradient flows, convergence to minimizers appears plausible, if not tautological. There is a plethora of work on asymptotic convergence analysis of gradient flows [7, 8]. In [8], the focus is on the exponential stability of the GF-based methods. The strong or strict convexity of the objective function is a standard assumption. In [9], it is shown that the condition can be relaxed by assuming that the objective function satisfies the Polyak–Łojasiewicz (PL) inequality, i.e., the objective function is gradient dominated. The following Lemma summarizes the classical results related to gradient flow present in the literature.

Lemma 1.1 *[10–12] Let $f : \mathbb{R}^n \to \mathbb{R}$, be continuously differentiable with locally Lipschitz* *derivative* $\nabla f : \mathbb{R}^n \to \mathbb{R}^n$ *such that for some* $l \in \mathbb{R}$*, the sublevel set* $f^{-1}(l) = \{\zeta \in \mathbb{R}^n | f(\zeta) \leq l\}$ *is compact. Then, the following statements hold for the gradient flow* $\dot{\zeta} = -\nabla f(\zeta)$*:*

1. *Trajectories starting in* $f^{-1}(l)$ *converge to the set of critical points, i.e., points* ζ^* *such that* $\nabla f(\zeta^*) = 0$*.*
2. *If* f *is analytic or convex, then every solution starting in* $f^{-1}(l)$ *converges to a single point.*
3. *Every asymptotically stable equilibrium is a strict and isolated minimizer, and every local minimizer is stable. If* f *is analytic or convex, then the set of all (strict) minimizers is equivalent to the set of (asymptotically) stable equilibria.*

Unconstrained Optimization: Finite-Time/Fixed-Time Convergent Gradient Flows

Consider the following unconstrained minimization problem

$$\min_{\zeta \in \mathbb{R}^n} g(\zeta) \tag{1.1}$$

where $g : \mathbb{R}^n \to \mathbb{R}$. The following assumption is necessary for the convergence of gradient-based methods to an optimal solution.

Assumption 1.1 *There exists* $\zeta^* \in \mathbb{R}^n$ *such that* $-\infty < g^* = g(\zeta^*)$*.*

Lemma 1.2 *[13] If* g *is convex and continuously differentiable, then a point* ζ^* *is the global optimal point of the function* g *if and only if* $\nabla g(\zeta^*) = 0$*. If* g *is strictly convex, then the optimal point* ζ^* *is unique.*

In [14], the authors introduce the normalized and signed gradient flows associated with a differentiable function that have the property of convergence to critical points of a function in finite time. In [7], the authors discuss a rescaled accelerated gradient flow dynamics having the following dynamics:

$$\dot{\zeta} = -\frac{\nabla g(\zeta)}{\|g(\zeta)\|^{\frac{q-2}{q-1}}}, \tag{1.2}$$

where, $q > 2$.

Lemma 1.3 *[7] If $g : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable and strongly convex, then the trajectories of* (1.2) *converge to the optimal point ζ^* in finite time, for $q > 2$.*

Inspired from (1.2), authors in [5] modified GF dynamics, so that the optimal point of (1.1) can be obtained within a fixed time for any $\zeta(0) \in \mathbb{R}^n$. In [5], the authors propose the following fixed-time convergent GF approach:

$$\dot{\zeta} = -c_1 \frac{\nabla g(\zeta)}{\|\nabla g(\zeta)\|^{\frac{q_1-2}{q_1-1}}} - c_2 \frac{\nabla g(\zeta)}{\|\nabla g(\zeta)\|^{\frac{q_2-2}{q_2-1}}} \tag{1.3}$$

where, $q_1 > 2$, $1 < q_2 < 2$, and $c_1, c_2 > 0$.

Lemma 1.4 *[5] If the objective function g satisfies Assumption 1.1, then the trajectories of* (1.3) *converge to the optimal point ζ^* in a fixed time for all $\zeta(0)$.*

Convex Optimization With Linear Equality Constraints: Finite-Time/Fixed-Time Convergent Gradient Flows

Consider the following constrained optimization problem

$$\begin{aligned} &\min_{\zeta \in \mathbb{R}^n} g(\zeta) \\ &\text{subject to } C\zeta = d \end{aligned} \tag{1.4}$$

where, $\zeta \in \mathbb{R}^n, g : \mathbb{R}^n \to \mathbb{R}$ is convex and continuously differentiable, $C \in \mathbb{R}^{m \times n}$, and $d \in \mathbb{R}^m$. In [1], the authors propose the following signed projected gradient method to solve the optimization problem (1.4):

$$\dot{\zeta} = -\beta P^\top \text{sign}(P \nabla g(\zeta)), \tag{1.5}$$

where, $P \in \mathbb{R}^{n \times n}$ is called projection matrix, sign$(\cdot)$ is the signum function defined componentwise, and $\beta > 0$. Following the assumption that columns of matrix C are linearly independent, authors in [1] discuss the finite-time convergence analysis of the sign projected gradient system (1.5).

1.2.2 Time-Varying Optimization

Most optimization problems encountered in real-time applications—such as guidance and navigation of aerial and autonomous vehicles, machine learning, and economics—are dynamic in nature [15–17]. A TVO problem involves an objective function and/or constraints that depend on time. Gradient flows are modified to solve TVO problems. To address this problem in a practical setting, an algorithm should have the following two properties: convergence to the time-varying optimum and the ability to track it. In [18], the author shows that the gradient method is able to converge to an arbitrarily small neighborhood of the optimum, assuming strong convexity and twice differentiability of the objective function. In [19], a dynamics is proposed that is able to converge, in the sense that $\lim_{\zeta\to\infty} \frac{\partial g(t,\zeta)}{\partial t} = 0$, assuming objective function $g(t, \zeta)$ is twice differentiable with respect to ζ for all t. In the literature, commonly used techniques to solve TVO problems are online optimization [20, 21], batch algorithms [22], and prediction–correction methodology. Online optimization is an unstructured algorithm [22]. It estimates the optimal points at each time instant using only past information, without leveraging the underlying properties of the optimization problem. In Batch algorithms, objective functions and constraints are sampled, and at each instant corresponding sequence of time-invariant constrained optimization problems are solved. This approach ignores the dynamic aspect of the problem, since each iteration tends to converge toward the optimal point of the sampled time-invariant problem, while the solution of the TVO problem is drifting away over time. Whereas, prediction-correction methodology is a time-structured approach.

Prediction–correction algorithms are commonly employed to track the optimizer trajectories of time-varying convex optimization problems. The prediction correction algorithm assumes strong convexity of the objective function to solve unconstrained [23] and constrained [24] TVO problems. These methods consist of a “prediction” step to estimate the optimizer trajectory at the next time instant, followed by a “correction” step to adjust the predicted value toward the true optimizer path. A prediction–correction-based continuous time dynamical system approach to solve unconstrained TVO is proposed in [3, 25], and guarantees the asymptotic convergence of the trajectories of the proposed system to the optimizer trajectory. More precisely, they propose a continuous time dynamical system $\dot{\zeta} = f(t, \zeta(t))$ whose solution $\zeta(t)$ satisfies $\|\zeta(t) - \zeta^*(t)\| \to 0$ as $t \to \infty$ for all initial conditions. In [3], the authors propose a projected dynamical system approach for tracking the optimizer trajectory of a time-varying nonconvex optimization problem. In this method, inequality constraints are handled by projecting the solution onto the feasible region at each time instant. The inequality constrained TVO problems are solved in [3] using logarithmic barrier functions with appropriately chosen slack variables and barrier parameters. In [26], authors propose projected dynamical system approaches to track the optimizer trajectory of an inequality constrained convex optimization problem with time-varying objective function and time-varying inequality constraints, based on the Karush–Kuhn–Tucker (KKT) optimality conditions and the optimizer trajectory characterization derived from the KKT conditions. The asymptotic and fixed-time convergence of the proposed projected dynamical system to the optimizer trajectory is shown via the Lyapunov-based analysis. In [27], authors develop a projected dynamical system approach to track the optimizer trajectory of a TV nonconvex optimization problem, where the inequality constraints are eliminated by projecting the solution to the feasible region at each instant of time.

1.2.3 Data Driven Inference of Passivity properties

First-principles modeling is typically a time-consuming process that demands specialized expertise. However, as systems grow increasingly complex, model-based control approaches are reaching their limitations. Meanwhile, the volume of stored data continues to expand rapidly. Across various disciplines, researchers are encountering both challenges and opportunities presented by big data. In engineering, simulation and experimental data are now ubiquitous, often capturing all critical system properties. This presents control engineers with a new challenge—and an opportunity—to integrate vast amounts of data into the framework of systems and control theory. Many data driven control design methods are present in the literature, summarized in [28]. In general, these approaches give little insights into the system.

One of the alternate approaches is to analyze system properties, using data first and leverage this knowledge for controller design. More specifically knowledge of passivity properties allow for the application of various well-known feedback theorems for designing a controller with performance guarantees [29]. In [30], the authors develop a systematic approach to determine whether and, if so, to what extent a discrete linear time invariant system is passive, where no knowledge about the model is available except for input-output samples. Quantitative measures of passivity, such as the shortage of passivity, which is indicated using output feedback passivity (OFP) index, play an important role in systems analysis, stability studies, and controller design [31].

Consider the following physical system shown in Figure 1.1, depicted as $\mathfrak{H} : u \mapsto y$, where u is the input, and $y = \mathfrak{H}(u)$ denotes the output. The system $\mathfrak{H}$ is said to be

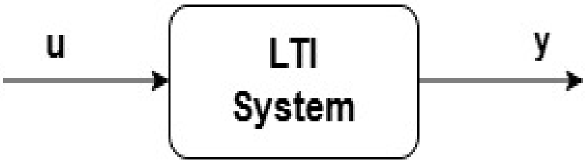

Figure 1.1: LTI system where we can choose the input u, and measure the output y

passive if $\langle \mathfrak{H}(u), u \rangle \geq 0$ for all possible inputs u, where u is taken from some Hilbert space of which $\mathfrak{H}$ is an endomorphism, and $\langle \cdot, \cdot \rangle$ denotes the inner product of that Hilbert space [32]. Quantitative measure of the passivity properties of a system are represented by passivity indices such as input feed forward passivity (IFP), and output feedback passivity (OFP). IFP corresponds to the minimum feedforward action required for rendering the system passive. OFP represents a shortage of passivity in a system [33]. In [30], authors formulated passivity indices as an optimal solution to an optimization problem for discrete-time LTI systems. They utilize gradient flows to calculate passivity indices.

To the best of the author's knowledge, this thesis is the first to address finite-time stabilization of a closed-loop system by leveraging the elegant feedback interconnection property of passive systems. Finite-time stabilization of discrete-time autonomous systems is recently been thoroughly studied in [34].

1.3 Contributions

This thesis stands as a notable contribution in advancing the field of continuous-time optimization, presenting an innovative approach through the introduction and application of dynamical systems-based on a predefined-time convergent gradient flows. The profound impact of this research extends across various dimensions, with key contributions manifesting in the following facets:

- At first, our investigation focused on continuous-time unconstrained optimization problems, leading to the development of a novel gradient flow with predefined-time convergence. This flow is rigorously designed for strongly convex objective functions and further extended to accommodate functions satisfying the more relaxed Polyak-Łojasiewicz (P-Ł) condition. Unlike existing methods, our scheme allows the selection of the upper bound on convergence time a priori—independent of the initial conditions or intricate parameter tuning. This significant simplification facilitates practical implementation while ensuring theoretical guarantees of convergence via Lyapunov-based analysis. Building upon this foundation, we integrated the concept of predefined-time stable gradient flows with projected gradient techniques to address convex optimization problems subject to linear equality constraints. As a result, we proposed a projected gradient flow algorithm with predefined-time convergence, offering the same initialization-free time-setting advantage. Furthermore, to address system perturbations, a perturbed version of the projected flow was analyzed within the input-to-state stability framework, enhancing robustness. The efficacy and practical relevance of the proposed algorithms were validated through numerical studies on linear least squares and resource allocation problems, with simulation results confirming their rapid and reliable convergence performance.

- Next, we extended the discussion on predefined-time gradient flows by involving the partial derivative of the gradient with respect to time, to solve TVO problems. We introduce a novel continuous-time dynamical system capable of tracking optimal trajectories for time-varying optimization problems subject to time-varying constraints, achieving convergence within a user-defined, predefined time frame. To address practical limitations, we develop a robust Newton-like algorithm that handles scenarios where the exact rate of change of the objective function's gradient is unavailable. Furthermore, for cases involving singular or near-singular Hessian matrices, we propose a Levenberg-Marquardt-like algorithm, ensuring reliable performance. Rigorous Lyapunov-based convergence analysis underpins the theoretical guarantees for both proposed algorithms. Comprehensive simulation studies demonstrate the efficacy and robustness of this predefined-time approach across diverse time-varying optimization problems.

- Further, this thesis delved into the demonstration of the proposed dynamics in real-world problems like collision-free robot navigation in complex sphere world environments. We develop a vector field-based navigation strategy defined over a compact, convex workspace punctured by disk-shaped obstacles. This strategy guarantees predefined-time convergence to a moving target position for a disk-shaped robot, starting from almost all initial conditions (excluding a set of measure zero) while ensuring strict collision avoidance throughout the trajectory. Crucially, we demonstrate that this navigation problem can be effectively formulated and solved as a Time-Varying Optimization (TVO) problem with inequality constraints. The efficacy and robustness of our proposed predefined-time convergent dynamics, along with its application to this challenging navigation task, are rigorously validated through comprehensive simulation results.

- Finally, our focus shifted to the design of controllers using the perspective of a continuous-time optimization framework. Hence, we explored the investigation of passivity properties of system using modified gradient flows, due to its relevence in controller design. We develop a novel methodology to determine passivity indices (input feedforward passivity - IFP and output feedback passivity - OFP) without requiring explicit system models. Our approach formulates passivity indices as an optimization objective and introduces prescribed-time gradient flows (GFs) that guarantee convergence within a user-defined time horizon. Crucially, this method operates solely on optimal input-output samples, computing necessary gradients directly from data. Furthermore, we establish how these data-derived passivity indices enable stabilization: By exploiting feedback interconnection properties, we design controllers (using IFP/OFP systems) that ensure both asymptotic and finite-time stability—all achieved without any knowledge of the underlying system model. This represents the first integration of prescribed-time optimization with passivity-based data-driven control, bypassing traditional modeling barriers. We successfully implemented the proposed controllers in the coupled tank system, presenting the outcomes through simulation results. The successful attainment of the desired water level serves as a testament to the efficacy of the proposed controller design approach.

1.4 Organization of the Thesis

This thesis comprises a total of seven chapters.

In the inaugural chapter of the thesis, Chapter 1 offers a comprehensive introduction to the conducted work. This includes a nuanced exploration of the motivation driving the research, an in-depth literature survey to discern the existing research gap, and a meticulous delineation of the principal contributions embedded within the thesis. This chapter not only establishes the foundation for the subsequent sections but also strategically defines the research objectives. Furthermore, it underscores the significance and relevance of the proposed work in the broader context of the academic and research landscape.

Chapter 2 serves as a comprehensive exploration of the mathematical preliminaries essential for grasping the intricacies presented throughout the thesis. Within this chapter, we delve into key notions, definitions, and concepts that lay the groundwork for a better understanding of the subsequent chapters. This encompassing overview includes critical topics such as comparison functions, and stability notions framed within the continuous-time systems paradigm. By delving into these foundational mathematical elements, this chapter serves as a pivotal resource, establishing the necessary framework for engaging with the advanced concepts and methodologies expounded upon in the subsequent sections of the thesis.

Chapter 3 introduces dynamical systems designed to solve unconstrained and equality-constrained optimization problems, ensuring convergence within a predefined time under the assumption that the objective functions satisfy the gradient dominance condition. The convergence analysis is carried out using Lyapunov-based methods. This chapter also examines the impact of perturbations on the proposed gradient flows through the lens of input-to-state stability theory. Furthermore, simulation results on least squares and resource allocation problems demonstrate the effectiveness of the proposed approach.

Chapter 4 presents a class of second-order dynamical systems designed to solve TVO problems with the capability to track the optimal solution trajectory within a predefined time horizon. Initially, a Newton-like dynamic is formulated for optimal trajectory tracking in constrained convex time-varying optimization problems. To address scenarios where the time derivative of the gradient is not explicitly available, a robust Newton-inspired dynamic with predefined-time stability is proposed. Furthermore, to handle situations involving non-invertible or singular Hessians, a Levenberg–Marquardt-type predefined-time stable dynamic is introduced.

Chapter 5 explores the real-world applications of the proposed dynamics in Chapter 4. The proposed approach is applied to robot navigation with collision avoidance, where the robot is required to track a moving target within a predefined time. By formulating the task as a TVO problem with inequality constraints, convergence to the moving target is ensured from almost all initial conditions. Simulation results confirm the robustness and effectiveness of the proposed dynamics in complex, obstacle-rich environments.

Chapter 6 applies the proposed gradient flow framework to analyze passivity properties and enable data-driven controller design. A novel method is introduced for computing passivity indices and achieving stabilization without relying on system models. By formulating passivity index computation as a predefined-time optimization problem, solved via gradient flows using input-output data, the approach enables model-free stability guarantees through feedback interconnections. This marks the first integration of prescribed-time optimization with passivity theory for model-free control.

Finally, the concluding chapter encapsulates the essential discoveries and contributions outlined in the thesis, offering a comprehensive overview. It delves into the implications of the research, acknowledges its limitations, and outlines potential paths for future research and practical applications. This chapter serves as the culmination of the thesis, providing a sense of closure to the overarching study.