Chapter 1

Introduction

Control theory enables the study and analysis of a system's behavior, with stability analysis playing a crucial role in understanding its qualitative characteristics. The rate of convergence indicates how quickly the system states reach the equilibrium point. Various relevant notions in this context include exponential stability, finite-time stability, fixed-time stability, and the more recently introduced prescribed-time stability. In practical systems, time constraints are frequently encountered. For instance, strict time bounds are evident in applications such as the interception of a target by a missile within a specified time frame, fault detection and correction within given time constraints, maneuvers of space launch vehicles that must be executed within specific time limits, and networked multi-agent systems that need to reach an agreement within a stipulated time. These scenarios highlight the importance of considering time factors when designing and analyzing control systems for real-world applications.

Classical asymptotic stability implies that the time required for the states of a system to converge to equilibrium is infinite. However, recognizing the need for control algorithms that ensure convergence within a finite time, researchers explored the concept of finite-time stability. Early examples of finite-time convergence were initially observed in classical optimal control [1]. However, these methods often involved discontinuous dynamics, which brought certain drawbacks like chattering. To address these issues, researchers turned to continuous time-varying feedback controllers to achieve finite-time convergence, as seen in [2]. Nevertheless, analyzing the stability of such time-varying systems presented its own challenges. Consequently, much attention was directed towards achieving finite-time convergence with continuous dynamics [3–8]. These efforts aimed to develop control techniques that guarantee convergence to equilibrium within a finite and predictable time frame, enhancing the applicability of control theory in various practical scenarios.

Later, researchers recognized the practical importance of having an upper bound on the convergence time, a feature lacking in existing finite-time control algorithms. This led to the proposal of fixed-time stability [9–11]. However, achieving this fixed convergence time required the estimation of an upper bound, which often proved to be challenging and parameter-dependent. Moreover, the estimated upper bounds tended to be conservative, limiting the potential benefits of this approach. Recently, the concept of predefined time stability was introduced to address these limitations [12]. This approach aimed to provide a more accurate and less conservative upper bound for convergence time estimation. However, it is worth noting that these algorithms were initially restricted to first and second-order systems only, which posed limitations since many practical applications involve systems with higher-order dynamics.

Indeed, fixed-time convergence algorithms can suffer from the drawback of requiring a substantial amount of control effort, which can limit their practical applicability. To make these algorithms more suitable for real-world scenarios, it becomes crucial to design control techniques that ensure the control demand remains bounded, preventing excessive control efforts. Furthermore, discontinuous control may not be desirable or feasible in all applications, making it necessary to explore continuous fixed-time algorithms. Continuous control methods offer smoother and more predictable responses, which can be advantageous in specific practical systems. A significant challenge with many existing finite and fixed-time algorithms is their intricate structure, which complicates their extension to higher-order dynamics. In order to tackle this issue, there is a need for intuitively clear and analytically simple approaches. Control techniques that are easy to understand and analyze can facilitate their application to systems with higher-order dynamics, making them more versatile and broadly applicable. Simplicity in algorithm design can lead to more efficient implementation and better performance in complex systems, enhancing their usability in various real-world applications.

Since most of the systems inevitably face varying system characteristics and operating conditions in some form, therefore, sophisticated control strategies that can adapt to changing conditions have become an important and desirable trait of modern control algorithms. In this regard, a number of approaches are available, for example, adaptive control, model reference adaptive control, self-tuning regulators and robust adaptive control etc. Notably, adaptive backstepping control, which falls under the umbrella of adaptive control techniques, has gained popularity due to its simplicity and adaptability [13]. The core idea behind backstepping control is to transform a complex nonlinear system into a series of interconnected subsystems, making it easier to design control laws for each subsystem sequentially. The adaptation mechanism within the controller allows the system to handle uncertainties and variations in the parameters, making it particularly suitable for applications where accurate system modeling is challenging or not feasible. Adaptive backstepping control finds applications in a wide range of fields, including robotics, aerospace, automotive systems, and industrial processes. Although adaptive backstepping control offers compelling advantages, its successful implementation requires careful consideration of several challenges. These challenges include the selection of appropriate adaptive laws, ensuring convergence of parameter estimates, and avoiding control signal chattering, which could cause undesirable oscillations in the system.

Multi-agent systems consist of interconnected entities like robots, sensor networks, and social networks. Consensus protocols enable these agents to reach collective agreement [14]. Finite-time and fixed-time consensus are intriguing extensions. Finite-time consensus aims to achieve agreement within a specific time frame, which is crucial for time-sensitive applications. It ensures rapid convergence to desired values. Fixed-time consensus goes further, imposing strict time constraints on convergence, irrespective of initial conditions. This guarantees predictable and fast decision-making.

Nonlinear optimal control is a control strategy used to optimize the performance of complex nonlinear systems. It involves finding control inputs that minimize a specified cost function while considering the system's nonlinear dynamics and constraints. Unlike linear optimal control, this approach caters to a wider range of nonlinear systems. In recent years, the demand for time-sensitive control strategies has grown significantly, driven by applications in robotics, autonomous systems, and real-time decision-making scenarios [15, 16]. Traditional optimal control techniques may not always suffice in these dynamic and high-speed environments, making nonlinear optimal finite time control an attractive alternative.

1.1 Motivation

Control systems play a vital role in numerous engineering applications, ranging from robotics and aerospace to industrial automation and autonomous vehicles. The objective of control systems is to manipulate the behavior of dynamic systems to achieve desired performance objectives. Traditional control techniques, such as PID controllers and model-based control, have been extensively studied and widely applied in various domains. However, as technology advances and systems become more complex, traditional control approaches face limitations in meeting the demands of modern applications.

One key challenge in control system design is the need for fast and accurate responses within specific time intervals. Many real-world systems require not only stable and accurate control but also control that can be achieved within prescribed time constraints. For instance, in unmanned aerial vehicles (UAVs), timely and precise control is crucial to ensure safe flight maneuvers and avoid collisions. Similarly, in manufacturing processes, control systems must meet tight timing requirements to optimize production efficiency and maintain quality standards. These examples highlight the need for control methodologies that can guarantee prescribed-time convergence, ensuring desired performance within pre-specified time intervals.

The motivation behind this thesis is to explore and develop prescribed-time control methodologies for nonlinear systems to address the limitations of traditional control techniques and meet the demands of modern applications. The thesis aims to provide novel approaches that enable timely and accurate control, even in the presence of uncertainties and performance requirements. By developing advanced control strategies that ensure prescribed-time convergence, this research seeks to enhance the capabilities and efficiency of various engineering systems.

One of the primary motivations for pursuing prescribed-time control is the increasing demand for high-performance control systems in critical applications. In domains such as aerospace, defense, and autonomous vehicles, the ability to achieve precise and timely control is paramount. For example, in UAVs used for surveillance or search and rescue operations, control systems must respond quickly to changing environmental conditions and execute maneuvers within strict time constraints. Prescribed-time control methodologies offer the potential to enhance the agility and responsiveness of these systems, enabling them to operate more effectively in dynamic environments.

Another motivation is the presence of uncertainties in real-world systems. Uncertainties, such as parameter variations, external disturbances, and modeling errors, are inherent in many systems and can degrade control performance. Traditional control techniques often struggle to handle uncertainties effectively, leading to suboptimal performance or even instability. Prescribed-time control methodologies offer a promising avenue for dealing with uncertainties by ensuring convergence within a specified time, regardless of their effects. By incorporating adaptation and robustness mechanisms, prescribed-time control strategies can provide more resilient control in the presence of uncertainties.

Moreover, prescribed-time control can have significant implications for safety-critical systems. In domains like transportation, healthcare, and nuclear power, control systems must not only meet performance requirements but also adhere to stringent safety standards. Prescribed-time control methodologies can provide guarantees on the maximum time required to achieve the desired performance, thereby reducing the risk of system failures or accidents. By enabling control actions within predetermined time limits, these methodologies can enhance the safety and reliability of critical systems.

Furthermore, prescribed-time control has the potential to optimize the efficiency of various engineering processes. In manufacturing and industrial automation, production systems often operate under tight time constraints to meet production targets and minimize costs. By ensuring prescribed-time convergence, control systems can optimize process parameters and reduce cycle times, leading to increased productivity and improved efficiency. Prescribed-time control methodologies can also be applied to energy systems, where fast and precise control is crucial for load balancing, grid stability, and renewable energy integration.

Lastly, the advancement of computing technologies and the availability of more powerful hardware have opened up new opportunities for implementing sophisticated control algorithms. Prescribed-time control strategies often require complex computations and algorithms that may have been computationally infeasible in the past. However, with the increasing computational capabilities of modern processors and the availability of specialized hardware, such as field-programmable gate arrays (FPGAs) and graphics processing units (GPUs), the implementation of prescribed-time control algorithms has become more viable.

1.2 Literature review

The field of control systems has witnessed significant advancements in recent years, with researchers exploring various methodologies to tackle the challenges posed by nonlinear systems and time constraints. This literature survey provides an overview of the existing research on prescribed-time control, adaptive backstepping control, multi-agent systems, nonlinear optimal control and related areas, highlighting the key contributions and trends in the field.

1.2.1 Prescribed-time control

Prescribed-time control is a relatively new area of research that aims to guarantee convergence within a specified time interval. The introduction of prescribed-time control for nonlinear systems, pioneered by Song et al. [17], has injected considerable energy into the realm of finite-time control. This concept has captured increasing attention from the control community, sparking numerous subsequent investigations in this essential field over the past few years (see [18] and references therein). Looking back at the development of finite-time control theory, we can trace its origins back to the 1960s when finite-time stability was proposed in [19], [20], and [21], initially applied to simple linear systems. The idea behind finite-time stability is that a system is considered stable if, starting from a bounded initial condition, all closed-loop signals remain bounded and its states converge to zero (or a residual set) within a specified finite time interval. It is crucial to distinguish between finite-time stability and asymptotic stability. While a system may exhibit finite-time stability without being asymptotically stable, and vice versa, asymptotic stability pertains to the system's behavior over an infinitely long time interval. On the other hand, finite-time stability is a more practical concept that enables the study of closed-loop system behavior over a finite, possibly short, time span. Therefore, it finds applications whenever it becomes necessary to ensure that the system states converge to a specific small threshold within a short time frame, for instance, to avoid saturation or the activation of nonlinear dynamics.

Over the past few decades, significant advancements have been made in finite-time control for dynamic systems, as evidenced by the works of various researchers [4, 6, 8, 22–24]. In the pursuit of achieving finite-time stability for high-order nonlinear systems, different approaches have been suggested, including the homogeneous approach, sliding mode control (SMC) terminal sliding mode, higher-order sliding modes, and adding a power integrator [25–29]. These methods aim to achieve convergence within a finite time, but their effectiveness depends on explicit knowledge of initial conditions, which can limit their practical applicability when such information is not readily available for the system under consideration.

To overcome this limitation, a new concept called fixed-time control has emerged [10, 30–36]. Fixed-time control utilizes odd-order plus fractional-order feedback to design various closed-loop system dynamics. One notable advantage of fixed-time control is that it allows for the estimation of an upper bound on the settling time without relying on any information about the system's initial conditions. This feature enhances the applicability of fixed-time control in cases where knowledge of the plant's initial states is limited or inaccessible.

Despite the advantages of fixed-time control in estimating settling times, there is a lack of a straightforward and direct relationship between the control parameters and the desired upper bound of the settling time. As a result, under fixed-time control, the settling time is often overestimated, leading to inaccurate assessments of system performance. In some cases, the overestimation can be hundreds or even thousands of times larger than the actual settling time. Furthermore, both for finite-time control and fixed-time control, the settling time is not a directly tunable parameter since it depends on other design parameters of the controller. This dependence complicates the process of achieving precise control over the settling time. To address these challenges and provide better control over the settling time without being affected by initial conditions or other design parameters, the predefined-time control approach has been proposed [12, 16, 37–39]. With predefined-time control, it becomes possible to preset the least upper bound of the settling time, allowing for more accurate and reliable control performance regardless of the system's initial conditions or other design choices [40].

In recent times, a classical idea that was originally developed for strategic and tactical missile guidance applications [41] has been revisited and extended to high-order nonlinear systems, giving rise to prescribed-time control. Prescribed-time control combines the benefits of finite-time control, fixed-time control, and predefined-time control while also providing the unique advantage of precise settling time presetting. The concept of prescribed-time control holds significant importance in numerous practical engineering applications where transient processes need to occur within specific time constraints. Such applications include missile guidance, multi-agent consensus, obstacle avoidance in robotic systems, and more [42–44]. By using prescribed-time control, these systems can achieve precise and reliable control over the settling time, meeting the time requirements for their specific tasks and ensuring efficient and safe operation in various scenarios. Indeed, one of the key advantages of prescribed-time control is its robustness to external disturbances. Even in the presence of external perturbations, the prescribed-time control approach can effectively maintain stability and achieve the desired settling time for the system. Another significant benefit of prescribed-time control is that the control input remains smooth throughout the transient process. This smoothness in control signals helps in avoiding abrupt changes or discontinuities, leading to more stable and predictable system behavior during the control process. Furthermore, prescribed-time control does not require any prior information about the upper bound of non-vanishing perturbations during the control design. This feature is particularly valuable because it eliminates the need for precise knowledge of disturbances, which can often be challenging to obtain or estimate in practical scenarios. The combination of robustness to disturbances, smooth control signals, and not depending on precise disturbance information makes prescribed-time control a promising and practical approach for various engineering applications, ensuring accurate and reliable control performance in the presence of uncertainties and disturbances.

1.2.2 Adaptive backstepping control

In most of the practical applications of engineering, it is quite challenging to design efficient control strategies that can overcome the uncertainty of nonlinear systems and guarantee the resulting closed-loop system performs satisfactorily. There have been significant attempts made in order to design more efficient control strategies for uncertain nonlinear systems liable to parametric uncertainties. One of the most promising methods for controlling a range of systems and processes has been demonstrated to be adaptive control [45]. By obtaining and utilizing information regarding a physical system's operation, the field of adaptive control technique seeks to enhance the closed-loop characteristics of an uncertain physical system [13, 46, 47].

The concept of backstepping was introduced for the creation of adaptive controllers at the start of the 1990s. Backstepping-based design technique is an iterative Lyapunov-based approach for a subclass of nonlinear dynamical systems, i.e. in strict feedback form [48]. In reality, this technique ensures global stability (or regional stability) and tracking qualities if the plant to be controlled belongs to the group of systems that may be transformed into the form of parametric-strict feedback. The backstepping design method's ability to give a systematic, step-by-step algorithm-based approach to designing stabilizing controllers is a key benefit. This approach allows for the systematic design of Lyapunov functions and feedback control algorithms [49, 50]. The findings in [6] and [8] have been expanded in a variety of systems, adaptive finite-time stabilization of dynamical systems with parametric uncertainty [51], adaptive transient specification based stabilization of nonlinear systems with uncertain parameters in finite-time [52], adaptive finite-time stabilization of nonlinear dynamical systems under time-varying actuator faults [53], command filter backstepping based finite-time stabilization [54]. Even though finite-time control techniques have undergone much research, their settling times are still influenced by the original circumstances. Furthermore, sometimes it is practically hard or even inaccessible to collect information of the system initial conditions in real applications, which makes it hard to estimate the settling time. After that, [55], [56], and [57] provided additional findings of adaptive fixed-time regulation for the linear as well as nonlinear dynamical systems. Even while the outcomes of fixed-time stabilization can accomplish the plant state regulation to desired point in the necessary amount of time by properly adjusting the parameters of designed control law, it will be very challenging to achieve convergence at random time.

1.2.3 Constrained feedback control

In various real-world applications, control systems often have to adhere to specific constraints, such as input saturation or safety limits. As mentioned in Remark 2.12, the time-varying gain-based prescribed-time control strategy ensures that the system states and the control signal converges to zero as the time approaches the designated terminal time. However, a potential concern arises due to the use of time-varying scaling functions in designing the virtual controllers. These scaling functions tend to approach infinity as time (t) approaches the terminal time (t_p), leading to significant growth of the virtual signal within a short period. Such rapid growth may result in system instability. To mitigate this issue, one can consider implementing a constraint control method.

The constraint control method has been extensively studied due to its effectiveness in handling constraint phenomena, particularly in avoiding state constraint violations. To address these challenges, researchers have extensively studied constraint control methods, aiming to effectively manage constraint phenomena and prevent state constraint violations in nonlinear systems. In [58], a nonlinear mapping method is investigated for nonlinear strict-feedback systems with full state constraints. The study explores the use of nonlinear mappings to address the challenges of full state constraints, providing insights into stabilizing the system while satisfying the imposed constraints. Researchers in [59, 60] present a method to handle time-varying constraints in control systems. This approach ensures that the control scheme adapts to changing constraints over time, providing robust performance and stability even in dynamic environments. The problem of input saturation in control systems is tackled through different schemes in the literature. The variable structure control method, as discussed in [61], is proposed to effectively manage nonlinear saturations, offering improved control performance under challenging saturation conditions. Additionally, the smooth input nonlinear function of control method described in [62] presents an alternative approach to handle the complexities of nonlinear saturations. In [63], the authors propose an adaptive practical fixed-time tracking control method for a class of strict feedback nonlinear systems. The method guarantees that the system adheres to prescribed boundary constraints during its operation, providing reliable and robust control performance. These methods have shown promising results in ensuring stable and reliable operation while satisfying the imposed constraints. Indeed, while significant progress has been made in constraint control methods for nonlinear systems, there are still several areas that warrant further research. In particular, the development of more effective and versatile constraint control strategies is essential, especially in scenarios with complex nonlinear dynamics and uncertainties, while ensuring prescribed-time convergence.

The barrier Lyapunov function technique has shown promise as an effective method for achieving prescribed performance in control systems [64, 65]. It offers several benefits, such as preventing undesirable system behavior caused by output nonlinearity [66] and ensuring constraint satisfaction [67, 68]. Additionally, [69] proposes a hyperbolic tangent function in conjunction with the barrier Lyapunov function method to restrict virtual control signals within a constrained region during system operation. However, the barrier Lyapunov function method does have certain limitations that require further investigation. One notable limitation is that the initial states of the system must satisfy the constraints imposed by the barrier function. If the initial states violate these constraints, the control scheme needs to be redesigned to ensure stability and constraint satisfaction. This constraint on initial states can be restrictive and may hinder practical implementation in some scenarios. Moreover, the barrier Lyapunov function method may face challenges in maintaining constraint satisfaction when affected by external uncertain factors. Uncertainties, such as parameter variations or disturbances, can influence the system's behavior and potentially lead to violations of the imposed constraints. Devising robust and adaptive strategies to handle uncertainties and ensure constraint satisfaction is an important area for future research.

1.2.4 Multi-agent systems

The consensus problem is a significant issue in research on multi-agent systems (MASs), where the objective is to design a suitable control law that can drive the system to achieve consensus on a common value. In recent years, extensive research has been conducted on various types of consensus, including leader-following, average consensus, and group/cluster consensus. The leader-following configuration involves one agent acting as the leader, whose behavior is unaffected by the dynamics of the followers. This mechanism is found in many biological systems and is considered energy-saving. In view of this, numerous researchers have examined this issue from different perspectives, relying on their understanding of graph theory, leading to several noteworthy findings [70, 71].

Earlier studies have focused on asymptotical consensus, which ensures that consensus is achieved as time tends to infinity [72,73]. However, recent attention has shifted towards finite-time consensus, which offers faster convergence rates and provides an upper bound on the settling time [74–76]. Nevertheless, the estimated upper bound of settling time (UBST) in finite-time consensus algorithms heavily relies on initial conditions and relevant parameters, limiting their applicability in certain cases [8]. This limitation arises because obtaining the initial values of system states in advance is not always feasible, and the UBST estimate may not be adjustable to desired values in certain situations [77]. In scenarios with strict time constraints, fixed-time consensus becomes more advantageous as it guarantees consensus within a finite time, regardless of initial conditions and system parameters [78, 79]. This property makes fixed-time consensus particularly attractive in safety-critical applications where timing constraints are crucial [10]. Furthermore, fixed-time consensus has been successfully applied to practical applications such as formation control of unmanned aerial vehicles, synchronization of power systems, and multi-robot coordination [80]. However, the estimated UBST in fixed-time consensus algorithms still depends on certain designed parameters with certain restrictions. Therefore, there are cases where the estimated UBST cannot be adjusted to a desired value [17, 44, 81].

Prescribed-time consensus is a crucial research problem in various practical applications where achieving a task within a specific time frame is essential. Despite its significance, limited research has been conducted on prescribed-time consensus, and most existing work has focused on systems with known or available dynamics in advance [82–86]. However, achieving consensus in uncertain nonlinear MASs poses greater challenges, despite having wide practical applications. Currently, the predominant methods for addressing unknown terms in nonlinear models involve approximating these terms using neural networks (NNs). By utilizing NNs, the unknown function can be approximated, assuming that the function's domain is contained within a compact set [87].

1.2.5 Optimal nonlinear feedback control

Optimal control theory plays a crucial role in achieving optimal system performance. When designing a controller for a nonlinear system, our primary concerns are stability and fast convergence. However, using a large control input can be costly and cause saturation of the actuators, leading to instability in the closed-loop system. To address these issues, various techniques have been developed to minimize control effort while maintaining stability margins [88–93]. One such technique is discussed in [94], which proposes an infinite-horizon optimal stabilization strategy for a nonlinear system with asymptotic stability. The general approach offers the groundwork for expanding linear quadratic control to nonlinear nonquadratic control issues. Currently, the maximum principle is the sole way to generate optimal finite-time controllers, which typically does not result in feedback controllers [15]. For achieving control input minimization while ensuring the stability of an uncertain nonlinear system, there are several attempts made to integrate the optimal controller with an integral sliding mode control (ISMC) [95]. In [96], sliding mode control theory is discussed in detail from the perspective of the optimization problem. In [97], the control problem for uncertain nonlinear systems is discussed where an optimal second-order SMC is used. In [98], a motion control problem for robotic manipulators is discussed based on an integral sub-optimal second SMC. A finite-time optimal control strategy for the affine nonlinear systems is obtained in [99], where sufficient conditions on optimality are obtained by using the Hamilton-Jacobi-Bellman equation. Alternatively, a partial-state stabilization by an optimal finite-time control strategy is introduced and all the sufficient conditions are derived in [100]. In [101], optimal SMC problem is discussed for under-actuated systems. In [102], authors have used SMC for missile autopilot systems where they have employed an optimal sliding surface. In [103], a discrete-time SMC is used for the PWM inverters based on optimal sliding surfaces. In [104], multi-rate output feedback based on an optimal discrete-time SMC is discussed. As an extension of [94], [99] studies the optimal finite-time stability and stabilization problem in this sense. The designed controllers are actually feedback controllers since the obtained results formulated on the framework established in [94]. In particular, a nonlinear, nonquadratic performance functional-based feedback control problem with an indefinite horizon is taken into account.

1.3 Contributions

This thesis makes several significant contributions to the field of control systems by developing and applying prescribed-time control methodologies for nonlinear systems. The key contributions of this research can be summarized as follows:

- Firstly, this thesis introduces a novel prescribed-time adaptive backstepping control methodology for a class of nonlinear systems under parameter uncertainties. By combining backstepping techniques with adaptive laws, the proposed control approach achieves prescribed-time convergence. Theoretical analysis and simulation studies validate the effectiveness and robustness of the methodology. This contribution enhances the existing body of knowledge in adaptive control and provides a systematic approach to achieve prescribed-time convergence.

- Secondly, this thesis extends the adaptive prescribed-time control framework to address the control challenges of an uncertain twin-rotor helicopter. A specific control strategy is developed to ensure prescribed-time convergence of the helicopter's state variables, enabling precise and timely maneuvering. Through simulation, the performance of the proposed control scheme is evaluated, demonstrating its applicability and effectiveness in the context of twin-rotor helicopter. This contribution advances the state-of-the-art in control techniques for aerial vehicles and provides practical solutions for improved control of such systems.

- To address the challenge of adaptive prescribed-time constrained feedback control, this thesis presents a prescribed-time constrained feedback control strategy for uncertain twin-rotor helicopters. By integrating prescribed-time control with constraints on system inputs, the proposed approach ensures safe and efficient operation even in the presence of parameter uncertainties. The design of control laws and constraint handling mechanisms, as well as the theoretical analysis, is established using direct Lyapunov stability theory. Through simulation and experimental studies, the effectiveness and robustness of the proposed control approach are demonstrated.

- Additionally, this thesis extends the prescribed-time adaptive control strategy for DC-DC boost converters. This control strategy aims to achieve fast and accurate regulation of the output voltage within a specified time frame. By adapting the control laws based on system feedback, the proposed approach addresses uncertainties and variations in system parameters, ensuring robust performance and stability. The research provides a comprehensive analysis of the stability and convergence properties of the proposed prescribed-time adaptive control strategy for DC-DC boost converters. Theoretical analysis and proofs are presented to establish the convergence guarantees and robustness of the control approach. Simulations and real-time experiments are carried out to validate the efficacy of the control approach in regulating the output voltage and achieving rapid convergence.

- Furthermore, distinct from traditional finite-time consensus control methods, this thesis presents an adaptive neural network-based prescribed-time consensus control algorithm for nonlinear multiagent systems. Radial basis function neural networks are employed to estimate the unknown nonlinearities within the system, enabling the control method to adapt flexibly to changing system conditions and uncertainties. The attainment of prescribed-time consensus and the boundedness of all closed-loop signals are ensured through a comprehensive analysis that utilizes both algebraic graph theory and the Lyapunov direct method. Simulation studies underscore the effectiveness and applicability of the proposed algorithm in the context of the nonlinear Kuramoto oscillator dynamic system.

- Finally, as an extension of the concepts presented in [16, 94, 99], this thesis addresses the challenge of optimal prescribed-time stabilization, specifically the task of determining a state-feedback control that minimizes specific performance criteria while simultaneously ensuring predefined-time stability of the closed-loop system. In particular, the thesis provides sufficient conditions for a controller to solve the optimal prescribed-time stabilization problem for a given system. These conditions revolve around a Lyapunov function that meets specific differential inequalities to ensure prescribed-time stability, and the steady-state Hamilton-Jacobi-Bellman equation for achieving optimality. Moreover, this outcome is applied to the realm of prescribed-time optimal feedback control for a specific class of affine dynamical systems. Lastly, as a case study, the prescribed-time optimal control of a coupled tank system is executed using the developed methodologies, and numerical simulations as well as experiments are conducted to illustrate their performance.

1.4 Organization of the thesis

The thesis contains nine chapters.
Chapter 1 of the thesis provides a brief overview of the work conducted, including the motivation behind the research, a detailed literature survey to identify the research gap, and the main contributions of the thesis. It sets the context for the subsequent chapters by defining the research objectives and outlining the significance of the proposed work.
In chapter 2, we provide a concise overview of the mathematical preliminaries used throughout the thesis. It covers key notions, definitions, and concepts necessary for understanding the subsequent chapters, including state space model, comparison functions, stability notions, and graph theory. This chapter establishes the foundational mathematical framework required to engage with the advanced concepts presented in the thesis.
Chapter 3 introduces a novel prescribed-time adaptive backstepping control methodology for a class of nonlinear systems. The chapter presents the theoretical framework, stability

analysis, and convergence properties of the proposed control approach. In addition, the effectiveness and superiority of the methodology are validated through simulation studies using a practical example of a single-link manipulator with a flexible joint.

In Chapter 4, the prescribed-time adaptive backstepping control methodology is extended to address the control challenges of the twin rotor helicopters. A specific control strategy is developed to ensure prescribed-time convergence of the helicopter's state variables. The chapter includes the controller design, stability analysis, and simulation studies to evaluate the performance of the proposed control approach in the context of twin rotor helicopter.

Chapter 5 starts with an introduction emphasizing the significance of designing prescribed-time constraint feedback control for uncertain twin-rotor helicopters. It proceeds to present the development of a tailored control scheme specifically designed to handle uncertainties and input constraints in real-world systems. The chapter includes a stability analysis to validate the stability properties of the control approach. Furthermore, simulation and experimental studies are conducted to assess the effectiveness and adaptability of the proposed control approach. The results and discussion section presents the findings from both simulation and experimental studies.

Chapter 6 focuses on the application of the prescribed-time adaptive control methodology to DC-DC boost converters. The introduction section highlights the motivation and significance of achieving fast and accurate output voltage regulation within a specified time frame. An adaptive control strategy is proposed, incorporating the mathematical framework, stability properties, and convergence guarantees. Theoretical derivations and proofs are provided to validate the effectiveness of the prescribed-time adaptive control strategy. The results and discussion section presents the findings from both simulation and experimental studies, providing further insights into the performance of the control approach.

In Chapter 7, the focus is on achieving prescribed-time adaptive neural consensus in uncertain nonlinear multi-agent systems. A novel consensus control approach is proposed, combining adaptive control and neural networks to achieve prescribed-time convergence and consensus among the agents. The chapter includes a comprehensive theoretical analysis, encompassing stability proofs and mathematical derivations, to validate the effectiveness and robustness of the control approach. Additionally, simulation studies are conducted to further validate the proposed control approach and provide practical insights into its performance in achieving prescribed-time convergence and consensus in uncertain nonlinear multi-agent systems.

Chapter 8 delves into the investigation of prescribed-time optimal control for a class of nonlinear systems. The chapter introduces the concept of optimal prescribed-time stabilization, which optimizes a performance criterion while satisfying prescribed-time constraints. It then extends this methodology to address optimal prescribed-time stabilization for nonlinear affine dynamical systems. To demonstrate the practicality of the proposed approach, the chapter applies it to the coupled tank system. Through simulation and experimental studies, the performance of the prescribed-time optimal control strategy is evaluated, assessing its ability to achieve the desired system behavior. Additionally, a comparison with existing literature is provided to highlight the advantages and contributions of the proposed methodology in the field of prescribed-time optimal control.

The final chapter summarizes the key findings and contributions of the thesis. It discusses the implications of the research, its limitations, and potential avenues for future research and applications. The chapter concludes the thesis and provides closure to the overall study.