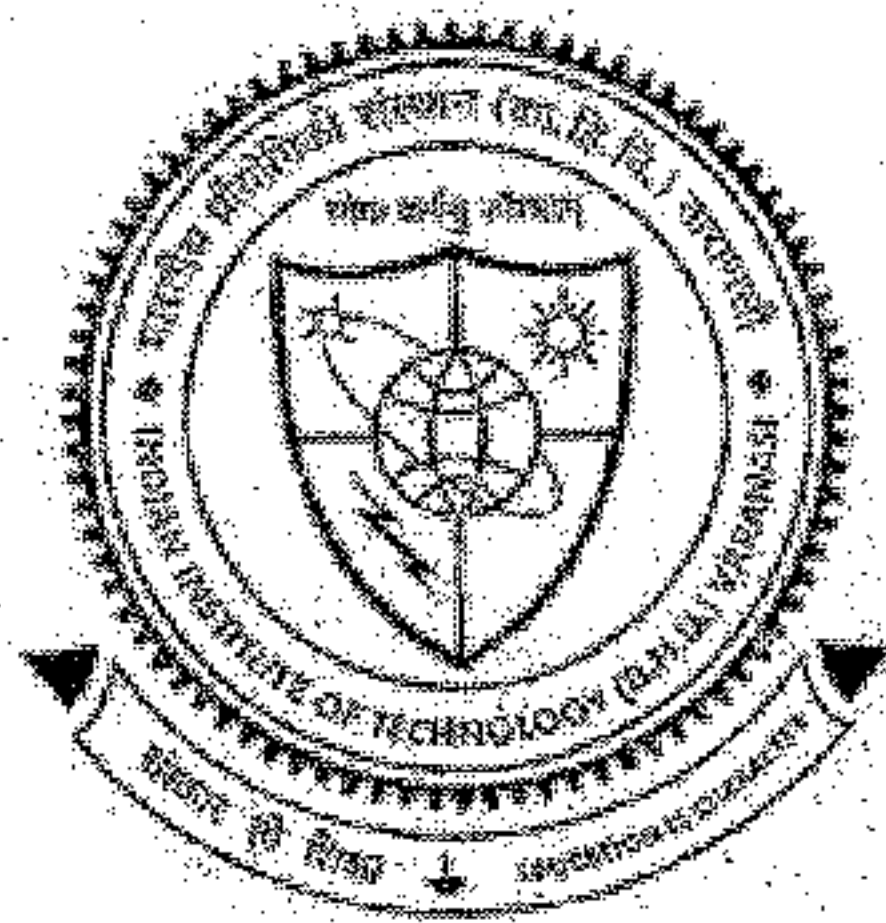


EXTENDED ABSTRACT

Mathematical Models for Treatment of Swallowing Disorders through Oesophageal Catheterisation



Name of student:

Anupam Kumar Pandey

Roll No.:

18121520

Degree for which

submitted:

Ph.D.

Department:

Mathematical Sciences

Signature of the Supervisor
(Prof. Sanjay Kumar Pandey)

ABSTRACT

This thesis investigates the impact of inserting a catheter in the oesophagus before surgical interventions by doctors. People suffer from various swallowing disorders based on different situations, such as problems with motility, structural abnormalities, neurological problems, and allergies that affect swallowing. These disorders become life-threatening if the oesophageal tissues cause abnormal growth inside. One of the most common reasons for this is gastroesophageal reflux. Various approaches, such as medical and pre-diagnosis using catheterization procedures, are not the best. Still, if required, one has to go for a surgical approach to providing a sign of relief to a patient. These real-life problems have been modeled mathematically in the thesis and thoroughly discussed to enlighten to whom they may concern. Seven chapters have been included in this thesis, each with its significance.

A brief introduction about physiological systems that resemble peristalsis in the oesophagus is discussed in **Chapter 1**. Swallowing disorders due to oesophageal diseases, such as sliding hiatus hernia and achalasia, and their signs and symptoms are properly elaborated. Given the pre-diagnosis, the purpose of modeling these problems using various approaches is briefly discussed. The solution approaches for the various fluid considerations, which are the required criteria to study the developed mathematical models, are mentioned.

Chapter 2 discusses a brief review of the literature of the earlier studies that have been done considering the peristaltic flow in different regimes. We present the requisite details about theoretical and experimental work on catheterization in oesophagus, a two-layered flow with Newtonian and non-Newtonian fluids. These studies led us to provide new insights into the field of mathematics and biomedical sciences. This chapter includes the main objective of our research work.

In **Chapter 3**, a mathematical model of a two-layered flow in a catheterized oesophagus is dealt with given peristaltic waves of dilating amplitude. This model provides new insight into the study of various swallowing disorders. The considerations of the two-layered model emphasize the interface developed between these layers. Further, the effect of viscosity on the interface in the presence of a catheter in this study provides a better estimate for a multilayered concept. In addition, preeminence is given to the pressure distribution throughout the oesophageal lumen with broadening of the catheter size. Due to the significant role of secondary peristalsis, feeding a patient with any liquid when a catheter is introduced may not be further suggested.

A mathematical model is developed to study the case of a sliding hiatus hernia a patient suffers without noticing it in **Chapter 4**. The presence of a sliding hiatus hernia makes the shape of oesophagus non-uniform. This study aims to unveil the problem of diagnosing the effects of catheterization in the exponentially diverging oesophageal tube. The effects of different characteristics, such as the broadening of the catheter, dilation, and gradient parameters, are investigated for the Newtonian flow regime. Moreover, due to the consideration of two-layered flow, the influence of the peripheral layer thickness on pressure and velocity with the volume flow rate is briefly discussed.

Chapter 5 discusses the non-Newtonian flow in the core of a two-layered model. Due to the inclusion of micropolar parameters and coupling numbers in this study, the model gets different aspects of fluid dynamics. This consideration affects the fluid flow due to its microrotation and microstructure model. This mathematical model discusses the impact of these characteristics on catheterization into the oesophagus. The micropolar fluid model caught the attention of various mathematicians and scientists worldwide because it simulates fluids made up of particles floating in a viscous medium with random orientation. Also, due to its application in physiology, such as during food swallowing in oesophagus and blood vessels, this has to be modeled by surrounding Newtonian flow. The discussion on pressure distribution and comparing the pressure magnitude with Newtonian and non-Newtonian flow are thoroughly explained.

Achalasia, which translates to "failure to relax" in Greek, is a rare condition of unclear etiology characterized by a lower oesophageal sphincter (LES) failure to relax and a loss of oesophageal peristalsis. Understanding the swallowing mechanism and making an accurate diagnosis is crucial because the management of dysphagia frequently depends on its origin. Pneumatic balloon dilatation is one of the treatment possibilities; because of this, a mathematical model is developed to study the oesophagus with stricture, such as achalasia in **Chapter 6**. The insertion of a novel catheter for this purpose in the oesophagus is discussed, and a pictorial view of deflated and inflated situations is provided. Further, the discussion regarding the placement of a deflated balloon can be disturbed and inflated as per the situation demands, which is given thoroughly. In addition, the question about the pressure distribution pattern along the oesophageal length due to the presence of stricture is discussed. At what stages, i.e., insertion, inflation, deflation, or withdrawal, will the pressure get maximum or minimum incorporated?

A study of power-law fluid on which the impact of dilating forcing amplitude is studied in **Chapter 7**. Several mechanisms are discussed, such as evaluating pressure, velocity, and stream functions when an external force is applied to the wall's surface of an elastic

tube. In the presence of sinusoidal wave forcing, the change in pressure distribution due to the external forcing is studied. The transition from pseudo-plasticity to dilatancy of the fluid requires more pressure or not to propel the food boluses toward the cardiac sphincter, which is given briefly. The effect of the flow behavior index on the velocity profile is discussed thoroughly. The perturbation techniques are used to obtain the solution to discuss all of the above. The power-law fluid shows different behavior with changes in flow behavior index, such as shear thickening and thinning of the fluid. With this, an attenuation of the radial velocity profile is also discussed.

The thesis concludes with overall conclusions and further scope of study.

References

- [1] H. Kanai, M. Iizuka, K. Sakamoto, One of the problems in the measurement of blood pressure by catheter-insertion: Wave reflection at the tip of the catheter, *Medical and biological engineering* 8 (1970) 483–496. doi:<https://doi.org/10.1007/BF02477185>.
- [2] R. Roos, P. S. Lykoudis, The fluid mechanics of the ureter, *Journal of Fluid Mechanics* 46 (4) (1971) 625–630. doi:<https://doi.org/10.1017/S0022112071000752>.
- [3] L. Srivastava, V. Srivastava, On two-phase model of pulsatile blood flow with entrance effects, *Biorheology* 20 (6) (1983) 761–777. doi:<https://doi.org/10.3233/BIR-1983-20604>.
- [4] L. Back, Estimated mean flow resistance increase during coronary artery catheterization, *Journal of biomechanics* 27 (2) (1994) 169–175. doi:[https://doi.org/10.1016/0021-9290\(94\)90205-4](https://doi.org/10.1016/0021-9290(94)90205-4).
- [5] L. Back, E. Kwack, M. Back, Flow rate-pressure drop relation in coronary angioplasty: catheter obstruction effect, *Journal of Biomechanical Engineering* (1996). doi:<https://doi.org/10.1115/1.2795949>.
- [6] V. Srivastava, R. Rastogi, Blood flow through a stenosed catheterized artery: Effects of hematocrit and stenosis shape, *Computers and mathematics with applications* 59 (4) (2010) 1377–1385. doi:<https://doi.org/10.1016/j.camwa.2009.12.007>.
- [7] S. K. Pandey, G. Ranjan, S. K. Tiwari, K. Pandey, Variation of pressure from cervical to distal end of oesophagus during swallowing: Study of a mathematical model, *Mathematical Biosciences* 288 (2017) 149–158. doi:<https://doi.org/10.1016/j.mbs.2017.03.010>.

- [8] A. Medhavi, U. Singh, Peristaltic transport of a two-layered fluid in a catheterized tube, *Theoretical and Applied mechanics* 39 (4) (2012) 291–311. doi:<https://doi.org/10.2298/TAM1204291M>.
- [9] A. Zeeshan, A. Riaz, F. Alzahrani, A. Moqet, Flow analysis of two-layer nano/johnson–segalman fluid in a blood vessel-like tube with complex peristaltic wave, *Mathematical Problems in Engineering* 2022 (2022) 1–18. doi:<https://doi.org/10.1155/2022/5289401>.
- [10] J. G. Brasseur, S. Corrsin, N. Q. Lu, The influence of a peripheral layer of different viscosity on peristaltic pumping with newtonian fluids, *Journal of Fluid Mechanics* 174 (1987) 495–519. doi:<https://doi.org/10.1017/S0022112087000211>.
- [11] J. Kahrilas, S. Wu, S. Lin, P. Pouderoux, Attenuation of esophageal shortening during peristalsis with hiatus hernia, *Gastroenterology* 109 (6) (1995) 1818–1825. doi:[https://doi.org/10.1016/0016-5085\(95\)90748-3](https://doi.org/10.1016/0016-5085(95)90748-3).
- [12] C. Andolfi, A. Jalilvand, A. Plana, P. M. Fisichella, Surgical treatment of paraesophageal hernias: a review, *Journal of Laparoendoscopic & Advanced Surgical Techniques* 26 (10) (2016) 778–783. doi:<https://doi.org/10.1089/lap.2016.0332>.
- [13] P. J. Kahrilas, H. C. Kim, J. E. Pandolfino, Approaches to the diagnosis and grading of hiatal hernia, *Best practice and research Clinical gastroenterology* 22 (4) (2008) 601–616. doi:<https://doi.org/10.1016/j.bpg.2007.12.007>.
- [14] S. Roman, P. J. Kahrilas, The diagnosis and management of hiatus hernia, *BMJ* 349 (2014). doi:<https://doi.org/10.1136/bmj.g6154>.
- [15] H.-X. Yu, C.-S. Han, J.-R. Xue, Z.-F. Han, H. Xin, Esophageal hiatal hernia: risk, diagnosis and management, *Expert review of gastroenterology & hepatology* 12 (4) (2018) 319–329. doi:<https://doi.org/10.1080/17474124.2018.1441711>
- [16] S. K. Pandey, S. Chandra, Transport of micro-polar fluids in an exponentially diverging tube by means of peristaltic waves of dilating amplitude (2022). doi:<https://doi.org/10.1002/zamm.202000162>.
- [17] A. R. Rao, S. Usha, Peristaltic transport of two immiscible viscous fluids in a circular tube, *Journal of Fluid Mechanics* 298 (1995) 271–285. doi:<https://doi.org/10.1017/S0022112095003302>.
- [18] S. Usha, A. Ramachandra Rao, Peristaltic transport of two-layered

- power-law fluids, *Journal of Biomechanical Engineering* (1997).
doi:<https://doi.org/10.1115/1.2798297>.
- [19] N. T. Williams, Medication administration through enteral feeding tubes, *American journal of health-system pharmacy* 65 (24) (2008) 2347–2357.
doi:<https://doi.org/10.2146/ajhp080155>.
- [20] K. S. Mekheimer, M. E. Kot, The micropolar fluid model for blood flow through a tapered artery with a stenosis, *Acta Mechanica Sinica* 24 (6) (2008) 637–644.
- [21] Zidan, L. McCash, S. Akhtar, A. Saleem, A. Issakhov, S. Nadeem, Entropy generation for the blood flow in an artery with multiple stenosis having a catheter, *Alexandria Engineering Journal* 60 (6) (2021) 5741–5748.
doi:<https://doi.org/10.1016/j.aej.2021.04.058>.
- [22] U. Khanduri, B. K. Sharma, Hall and ion slip effects on hybrid nanoparticles (au-go/blood) flow through a catheterized stenosed artery with thrombosis, *Proceedings of the Institution of Mechanical Engineers, Part C:*
n, Theory of micropolar fluids, *Journal of mathematics and Mechanics* (1966) 1–18.
- [24] M. Feldman, L. S. Friedman, L. J. Brandt, Sleisenger and Fordtran’s gastrointestinal and liver disease E-book: pathophysiology, diagnosis, management, Elsevier health sciences, 2015.
- [25] B. T. Johnston, Oesophageal dysphagia: a stepwise approach to diagnosis and management, *The lancet Gastroenterology & hepatology* 2 (8) (2017) 604–609.
doi:[https://doi.org/10.1016/S2468-1253\(17\)30001-8](https://doi.org/10.1016/S2468-1253(17)30001-8).
- [26] Bhattacharyya, The prevalence of dysphagia among adults in the united states, *Otolaryngology–Head and Neck Surgery* 151 (5) (2014) 765–769.
doi:<https://doi.org/10.1177/0194599814549156>.
- [27] T. Wilkins, R. A. Gillies, A. M. Thomas, P. J. Wagner, The prevalence of dysphagia in primary care patients: a hamesnet research network study, *The Journal of the American Board of Family Medicine* 20 (2) (2007) 144–150.
doi:<https://doi.org/10.3122/jabfm.2007.02.060045>.

- [28] J. Solt, J. Bajor, M. Moizs, E. Grexa, P. O. Horvath, Primary cricopharyngeal dysfunction: treatment with balloon catheter dilatation, *Gastrointestinal endoscopy* 54 (6) (2001) 767–771. doi:<https://doi.org/10.1067/mge.2001.118442>.
- [29] T. Young, Hydraulic investigations, subservient to an intended croonian lecture on the motion of the blood, in: *Abstracts of the Papers Printed in the Philosophical Transactions of the Royal Society of London*, 1, The Royal Society London, 1832, pp. 298–300. doi:<https://doi.org/10.1098/rspl.1800.0163>.
- [30] A. Y. Wong, P. Rautaharju, Stress distribution within the left ventricular wall approximated as a thick ellipsoidal shell, *American heart journal* 75 (5) (1968) 649–662. doi:[https://doi.org/10.1016/0002-8703\(68\)90325-6](https://doi.org/10.1016/0002-8703(68)90325-6).
- [31] H. Sandler, H. T. Dodge, Left ventricular tension and stress in man, *Circulation research* 13 (2) (1963) 91–104. doi:<https://doi.org/10.1161/01.RES.13.2.91>.
- [32] D. Takagi, N. Balmforth, Peristaltic pumping of viscous fluid in an elastic tube, *Journal of Fluid mechanics* 672 (2011) 196–218. doi:<https://doi.org/10.1017/S0022112010005914>.
- [33] S. K. Pandey, A. Singh, Peristaltic transport in an elastic tube under the influence of dilating forcing amplitudes, *International Journal of Biomathematics* 13 (04) (2020) 2050027. doi:<https://doi.org/10.1142/S1793524520500278>.
- [34] J. Misra, S. Pandey, A mathematical model for oesophageal swallowing of a food-bolus, *Mathematical and computer modelling* 33 (8-9) (2001) 997–1009. doi:[https://doi.org/10.1016/S0895-7177\(00\)00295-8](https://doi.org/10.1016/S0895-7177(00)00295-8).