

# Contents

List of Figures	x
List of Tables	xiv
List of Symbols	xiv
List of Abbreviations	xvii
Preface	xviii
<b>1 Introduction</b>	<b>1</b>
1.1 Interval-valued optimization . . . . .	4
1.1.1 Interval-valued function . . . . .	5
1.1.2 Formulation of interval optimization problem . . . . .	5
1.1.3 Preliminaries . . . . .	7
1.1.4 Literature survey . . . . .	15
1.2 Set-valued optimization . . . . .	18
1.2.1 Set-valued function . . . . .	21
1.2.2 Set optimization problem formulation and its solution concept . . . . .	22
1.2.3 Basic definitions . . . . .	23
1.2.4 Methods of solving set optimization problems . . . . .	26
1.2.5 Literature survey . . . . .	28
1.3 Motivation, objective and contribution of the thesis . . . . .	33
1.4 Organization of the thesis . . . . .	35
<b>2 Normal and Tangent Cones for Set of Intervals and Their Application in Optimization with Functions of Interval Variables</b>	<b>37</b>
2.1 Introduction . . . . .	37
2.2 Motivation . . . . .	38
2.3 Contributions . . . . .	38

2.4	Normal cone for a set of intervals . . . . .	39
2.5	Tangent cone for a set of intervals . . . . .	46
2.5.1	Polarity between tangent cone and normal cone . . . . .	53
2.5.2	Tangent cone characterization with the help of continuity of distance function in $I(\mathbb{R})^n$ . . . . .	55
2.6	Efficiency conditions employing normal cone . . . . .	57
2.6.1	$gH$ -differentiability of a function $\mathbf{T}: I(\mathbb{R})^n \rightarrow I(\mathbb{R})$ . . . . .	59
2.6.2	Optimality condition for constrained IOPs where domain is a subset of $I(\mathbb{R})^n$ . . . . .	62
2.6.3	An application . . . . .	66
2.7	Conclusion . . . . .	71
<b>3</b>	<b>Generalized Hukuhara Weak Subdifferential and Its Application on Identifying Optimality Conditions for Nonsmooth Interval-Valued Functions</b> . . . . .	<b>73</b>
3.1	Introduction . . . . .	73
3.2	Motivation . . . . .	73
3.3	Contributions . . . . .	74
3.4	$gH$ -weak subdifferential calculus for IVFs . . . . .	75
3.5	Optimality for the difference of two IVFs . . . . .	91
3.6	$gH$ -directional derivative and $gH$ -weak subdifferential for IVFs . . . . .	96
3.7	$\mathcal{W}$ - $gH$ -weak subgradient method . . . . .	100
3.7.1	Convergence analysis of $\mathcal{W}$ - $gH$ -weak subgradient algorithm . . . . .	107
3.8	Conclusion . . . . .	110
<b>4</b>	<b>Nonsmooth Interval Optimization with <math>gH</math>-Dini Hadamard Subdifferential and Its Applications in Control Systems</b> . . . . .	<b>111</b>
4.1	Introduction . . . . .	111
4.2	Motivation . . . . .	112
4.3	Contributions . . . . .	112
4.4	$gH$ -Dini Hadamard subdifferential and superdifferential for IVFs . . . . .	113
4.5	Optimality conditions by $gH$ -Dini Hadamard subdifferential . . . . .	126
4.6	An application of $gH$ -Dini Hadamard subdifferential . . . . .	131
4.7	Conclusion . . . . .	141
<b>5</b>	<b>Sufficient Optimality Conditions and Duality for a Nonsmooth Interval- Valued Optimization Problem with Generalized Convexity via <math>gH</math>-</b>	

<b>Clarke Subgradients</b>	<b>143</b>
5.1 Introduction . . . . .	143
5.2 Motivation . . . . .	144
5.3 Contributions . . . . .	144
5.4 $gH$ -Clarke subdifferential and $gH$ -generalized convex IVFs . . . . .	145
5.5 Optimality conditions for a nonsmooth optimization problem . . . . .	150
5.6 Duality theory for NIOP . . . . .	153
5.7 Conclusion . . . . .	160
<b>6 Trust-Region Method for Set Optimization Problems with Set-Valued Mapping Being Finitely Many Vector-Valued Functions</b>	<b>161</b>
6.1 Introduction . . . . .	161
6.2 Motivation . . . . .	162
6.3 Contributions . . . . .	162
6.4 Auxiliary concepts to solve set optimization problem . . . . .	163
6.5 Trust-Region method for set optimization . . . . .	168
6.5.1 Choice of ' $a^k$ ' from $P_k$ . . . . .	169
6.5.2 Choice of the trust-region step ' $s_k$ ' . . . . .	171
6.5.3 Calculation of reduction ratio . . . . .	173
6.5.4 Trust-Region radius update . . . . .	176
6.5.5 Stopping condition . . . . .	178
6.5.6 Well-definedness of Algorithm 3 . . . . .	183
6.6 Global convergence analysis . . . . .	183
6.7 Numerical experiment . . . . .	197
6.8 Conclusion and future directions . . . . .	211
<b>7 Non-Monotone Trust-Region Methods for Set Optimization with Finitely many Vector-Valued functions as Set-Valued mapping</b>	<b>215</b>
7.1 Introduction . . . . .	215
7.2 Motivation . . . . .	216
7.3 Contributions . . . . .	216
7.4 Non-monotone trust-region method for set optimization . . . . .	217
7.4.1 Well-definedness of Algorithm 4 and Algorithm 5 . . . . .	224
7.5 Global convergence analysis . . . . .	224
7.6 Numerical Analysis. . . . .	242
7.7 Conclusion and future directions . . . . .	247

<b>8 Conclusion and future directions</b>	<b>249</b>
8.1 Future scopes of research . . . . .	250
<b>9 Appendix A</b>	<b>254</b>
9.1 Proof of Lemma 1.1 . . . . .	254
<b>10 Appendix B</b>	<b>255</b>
10.1 Proof of Lemma 1.2 . . . . .	255
10.2 Proof of Lemma 1.3 . . . . .	255
10.3 Proof of Lemma 1.4 . . . . .	256
10.4 Proof of Lemma 1.5 . . . . .	257
10.5 Proof of Lemma 3.1 . . . . .	259
<b>11 Appendix C</b>	<b>260</b>
11.1 Proof of Lemma 1.6 . . . . .	260
11.2 Proof of Lemma 1.7 . . . . .	262
11.3 Proof of Lemma 1.8 . . . . .	264
11.4 Proof of Lemma 1.9 . . . . .	264
<b>12 Appendix D</b>	<b>266</b>
<b>References</b>	<b>275</b>
<b>List of Publications</b>	<b>293</b>