Chapter 7

Conclusion and future directions

In the previous chapters, successful attempts to solve MOPs were presented. In this chapter, the significant contributions of this thesis are summarized. The possible directions for future work are also discussed.

7.1 General conclusions

The principal conclusions of this thesis are as follows.

- Several decomposition based methods for solving MOPs with twice continuously differentiable objective and constraint functions are developed. We have used two effective scalarization techniques (cone method and modified Pascoletti-Serafini method) to decompose the MOPs into the single objective optimization subproblems. As the nature of the obtained subproblems were complex, we have applied interior-point methods (that are based on line search and trust-region techniques) to solve them.

- In each proposed method, we have introduced different merit functions and used them to check whether the new iterate is acceptable or not. Also, we have proved that the computed Newton directions descent for the proposed merit functions.

Moreover, for line search-based IPMs, we have utilized them to find the suitable step length along the search directions.

- The pseudo codes of each proposed method have been provided. Also, the convergence results have shown of the proposed methods.

- We experimentally evaluated the proposed algorithms in Chapter 2 and 3 using many performance metrics such as hypervolume (HV), modified generational distance (GD+), and inverted generational distance (IGD). Based on these metrics we compare the proposed algorithms with some popular existing solvers. The proposed methods are found to be highly competent compared to other existing techniques (see Chapter 2 and Chapter 3).

- The methods proposed in Chapter 4 and 5 are applied to the real-life optimal control problems. These problems are the system of differential equations. To solve them, we first converted them into bi-objective optimization problems and then applied the proposed methods of the respective chapters.

7.2 Contribution of the thesis

In the first contribution, reported in Chapter 2, we developed an infeasible interior-point method to solve convex and nonconvex MOPs. This method is derived with respect to the parametric single objective optimization problem that is obtained with the help of the cone method. Initially, we built an algorithm to deal with the convex case of the parametric single objective optimization problem, and later we extended it to solve the nonconvex case. The convergence analysis of the method and an estimate of the number of iterations to reach an ϵ-precise solution are also provided. We also discussed three strategies of choosing starting points and compare them with respect to the number of iterations and time. We provide the performance of the proposed methods on a variety of convex and nonconvex multiobjective test problems. Performance comparison between the proposed method and popular existing solvers is provided with respect to two performance measures and the corresponding relative efficiency measures.

In the next contribution, reported in Chapter 3, an interior-point approach has introduced to find a subset of nondominated points of an MOPs with the help of the cone method (IC-IPM). In proposed method, IC has been used to transform an MOP into a collection of single objective optimization problems. Each single objective optimization problem of the collection has been solved by IPM. To find the solution of each subproblem by IPM, a barrier problem and its KKT conditions have been derived. In order to the solve KKT conditions, the iteration started with the initial point and then calculated the direction by Newton method. Thereafter, step length has been chosen so that the nonnegative variables remain nonnegative. A new merit function has been also proposed with global properties. Merit function has helped to take a suitable step length along the search direction. Theorem 3.5 shows that the search direction calculated by Theorem 3.1 is descent for the merit function (3.17). Furthermore, we have proved the global convergence of the proposed algorithm under standard assumptions. We demonstrate through numerical experiments that Algorithm 4 can solve constrained MOPs efficiently. We have used three performance measures (GD+, HV, IGD) to test the efficiency of the Algorithm 4 on some standard test suite. The performance of Algorithm 4 has compared with some existing popular algorithms. Table 3.3 and 3.4 has shown that Algorithm 4 is comparatively efficient.

In another contribution, reported in Chapter 4, a primal-dual interior-point approach (PD-IPM) has introduced to solve the MOPs with the help of the cone method. In the proposed method, cone method has been used to transform an MOP into a collection of single objective optimization problems. Each single objective optimization problem of the collection has been solved by PD-IPM. To find the solution of each subproblem by PD-IPM, a barrier problem and its KKT conditions have been derived. In order to the solve the pertaining KKT systems, the iteration started with a given initial point and then calculated the direction by Newton method. Thereafter, step length has been chosen so that the nonnegative variables remain nonnegative. In the sequel, we have used a merit function to take a appropriate step length along the search direction. A sufficient condition for a point to be stationary of the used merit function has been also provided (Theorem 4.1). It has been found that the chosen primal directions in the movement of Algorithm 5 are descent directions of the used merit function (Theorem 4.2). To show the diversity of the proposed method, we applied the proposed method to solve an optimal control problem of carbon dioxide emission from energy sector with the efficiencies of mitigation options to reduce energy consumption rate and the CO_2 emission rate as their control variables. For which we proposed a multiobjective optimization approach to find the optimal strategies to minimize both—the CO_2 level from energy sector and the implementation cost of the control strategies. The result obtained by applying Algorithm 5 to the problem (5.32) has been analyzed for two different values of growth rate of energy use, i.e., $\gamma = 0.06$ and 0.1. We have obtained the trade-off curves for two values γ which shows that the difference between the best and worst case scenario for η_1 increases when γ increases. We have also observed that the CO_2 concentration with control strategy is significantly lower than without control strategy by the end of 21st century, regardless the values of γ (growth rate of energy use).

In Chapter 5, multi-objective optimization problems have been solved with the help of Pascoletti-Serafini scalarization technique and interior point method. The Pascoletti-Serafini scalarization has been used to transform an MOP into a set of scalar optimization subproblems. Thereafter, these subproblems have been solved with the help of trust region interior point technique. The performance of Algorithm 6 has been tested on some standard test problems. The results in Section 5.9 has been shown the efficiency of Algorithm 6. As an application, we have applied the proposed Algorithm to solve a model problem for tuberculosis from optimal control view point, using a multiobjective approach. The optimal control strategies are determined by simultaneously minimizing the number of individuals infected with tuberculosis and the cost of implementing prevention and treatment policies. Using the proposed approach, additional weight coefficients are not needed in order to formulate a single cost functional. The results of this study demonstrate how a comprehensive multiobjective approach can effectively identify the best control strategies within a mathematical model for tuberculosis. We found alternative viewpoints on implementing prevention and treatment policies using the obtained trade-off solutions. We analyze the optimal control strategies with varying β (transmission coefficient). The obtained results depict that as the transmission coefficient increases the proportion of active infectious and persistent latent individuals also increases. Furthermore, we have observed that the maximum and minimum values of f_1 increase with the increase of transmission coefficient β.

In Chapter 6, we have developed a new trust-region method for unconstrained MOPs. By using an approximation of the steepest descent direction, new local quadratic models have presented. Using the BFGS updating formula for the Hessian approximation of the models, we have shown that the proposed algorithm is convergent under some mild and standard conditions on the objective functions. We also implemented the proposed algorithm in MATLAB and applied on some nonsmooth examples.

Next, the possible future scope of the above-discussed work is discussed.

7.3 Future directions

The above-discussed contributions in the thesis may lead to several future research directions:

(i) Since interior-point methods are known to work well in higher dimensions and with structured or sparse problems, our future work will aim to rigorously investigate if the proposed IIPM (CM) will work well for high dimension and sparse multi-objective optimization problem.

(ii) To extend the efficiencey of the proposed algorithms, in future, we will consider nonsmooth MOPs with a nonsmooth merit function and apply inexact Newton methods.