CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter presents a brief overview of the literature from two viewpoints: (a) reviewing the literature regarding the development and relevant applications of the variational and upper bound rigid block method, and (b) reviewing the significance and development of the theory of unsaturated soil mechanics from the shear strength perspective and the application of the unsaturated shear strength formulations concerning the water content-suction stress relationships in addressing different unsaturated mechanics of soil in different stability problems.

2.2 LITERATURE REVIEW: VARIATIONAL METHOD AND UPPER BOUND RIGID BLOCK METHOD

In the following section, existing literature corresponding to the application of variational method, upper bound rigid block method in different stability problems is discussed consecutively.

2.2.1 Variational method

The application of variational method to stability problems in soil mechanics was originated by a group of scientists from the eastern countries of Europe. Gersevanov, the founder of Soviet school of soil mechanics was the first to employ variational method to solve problems of bearing capacity. In order to apply the variational technique to slope stability problems, Kopacsy (1961) was the first who discarded all presumptions about the character of the stress distribution along the slip surface and simply required that the distribution satisfy the three criteria of static equilibrium.

Kopacsy sought a slip surface where the soil's overall resistance to the action of a vertical load and its own weight was at its lowest. In spite of the fact that Kopacsy's work included several typos and a complex presentation, it was mostly ignored. Following his lead, a few researchers performed their own studies on stability problems using the variational technique, which is demonstrated below:

2.2.1.1 Goldshtein et al. (1969)

Goldshtein et al. (1969) investigated the problems of earth pressure on retaining walls using variational method. They selected the total pressure on the wall as the function to be optimized. The solution of an inclined, rough retaining wall was presented for a cohesive backfill with curvilinear free surface. The proposed algorithm was solved by means of a digital computer.

2.2.1.2 Kogan and Lupashko (1970)

Kogan and Lupashko (1970) was examined the stability of slopes. The factor of safety was calculated in their study as the ratio of the actual slope parameters to those of the slope at the limiting equilibrium. The actual slope parameters were determined experimentally and hence are sensitive to a variety of independent incidental factors such as soil homogeneity at sampling locations, moisture content variation, equipment inaccuracy, measurement errors, and so on. To correct these erroneous quantities, they used mathematical statistics to create some normal distribution curves.

2.2.1.3 Narayan (1975)

Narayan (1975) analyzed the stability of slopes as a minimization problem and obtained the critical slip surface which corresponds to minimum factor of safety satisfying all equilibrium and boundary conditions. The functional representing the factor of safety was the same as that used by Goldshetein (1969). Approximate solutions were obtained by the use of computer and employing some numerical techniques.

2.2.1.4 Revilla and Castillo (1977)

Revilla and Castillo (1977) employed variational method to the problem of slope stability. Using Janbu's model for homogeneous and frictionless rectilinear slope, the approach enables for identifying the critical sliding line (the one with the lowest factor of safety). This technique saves a significant amount of time and effort. Additionally, it provides the slope's actual safety factor, resulting in a more thorough understanding of the slope's safety.

2.2.1.5 Baker and Garber (1978)

Baker and Garber (1978) used a limit equilibrium approach to apply variational calculus. By satisfying all equilibrium conditions, the critical slip surface was found. They discovered that the critical slip surface is log-spiral for rotational failure and straight line for translational failure in the homogeneous and isotropic cohesive-frictional soil slope without pore water pressure or external loads.

2.2.1.6 Castillo and Luceno (1982)

Several variational approaches in slope stability analysis were criticised by Castillo and Luceno (1982). Baker and Garber's functional is unbounded, and hence the problem is presented incorrectly. Since Castillo and Revilla's technique is based on Janbu's model, it does not satisfy all of the equilibrium equations.

2.2.1.7 Leshchinsky and Reinschmidt (1985)

Leshchinsky and Reinschmidt (1985) investigated the stability of membrane reinforced slopes. He noticed that (i) the stronger the membrane, the deeper the failure, (ii) the elevation of the membrane has little effect on the stability or location of the slip surface if the failure passes through it, (iii) the presence of a membrane increases the compressive stress over the critical slip surface, and (iv) the presence of a membrane decreases the tensile stress that tends to develop near the crest.

2.2.1.8 Dixit and Mandal (1992)

Dixit and Mandal (1992) used the variational approach to investigate the bearing capacity of geo-synthetic reinforced soil. They observed that (i) the critical rupture surface has a log spiral shape, (ii) the shape of the critical rupture surface depends on both cohesion and the angle of internal friction, (iii) the approach is only valid for shallow reinforcement, and (iv) the bearing capacity increases significantly with the inclusion of reinforcement.

2.2.1.9 Leshchinsky and San (1994)

Leshchinsky and San (1994) incorporated a horizontal seismic coefficient to Baker and Garber's variational technique (1978). This formulation is based on the limit equilibrium method rather than the plasticity law. It calculates the minimum factor of safety for a log-spiral mechanism in which all static equilibrium equations are satisfied without making any assumptions in advance.

2.2.1.10 Ling et al. (1997)

Ling et al. (1997) observed the stability and permanent displacement of a slope subject to combined horizontal and vertical accelerations. A log-spiral failure mechanism was employed. Observations of this study indicate that the presence of a significant horizontal acceleration causes the permanent displacement calculation to be affected by vertical acceleration as well.

2.2.1.11 Baker (2003)

Baker (2003) employed variational method in the frame work of limiting equilibrium analysis, which involves identifying the critical slip surface and its associated minimal safety factor of slope. The study was carried out to establish the conditions necessary to guarantee the existence of a physically significant minimum safety factor in all slope stability formulations that satisfy equilibrium conditions without prior assumptions about the shape of potential slip surfaces. The study proposed three physical elements that must be considered to ensure the existence of a minimum safety factor. These include: (a) two integral inequality constraints that restrict acceptable forms of slip surfaces and normal stress functions, based on the observation that slopes typically fail by moving down and away from the main body of the slope. (b) The Mohr envelope strength model should indicate a finite tensile strength. (c) A "cracking criterion" should specify the consequences, such as crack formation, when the soil's tensile strength is fully mobilized. The aim of the study was to provide a sound basis for the limiting equilibrium technique used in engineering assessments of slope stability.

2.2.1.12 Baker et al. (2006)

Baker et al. (2006) used pseudo-static approach for analyzing slope stability under seismic conditions, which is commonly recommended in design manuals and codes. The unusual situations such as liquefaction, significant strength loss, and exceptionally large earthquakes were not taken into account in the study. The work used a variational formulation of limiting equilibrium and a strength reduction procedure to avoid kinematic and static assumptions. The purpose of the work was to present a design chart for pseudo-static analysis of homogeneous slopes that can be applied to a wide range of input parameters.

2.2.1.13 Chen et al. (2016)

Chen et al. (2016) discussed a theoretical framework for analyzing seismic active earth pressure on gravity retaining walls using the limit equilibrium theory and the variational method. The aim of the study was to determine the minimum and maximum values of seismic active earth pressure, which can help inform aseismic design. The approach in Chen et al. (2016)'s study involved using the isoperimetric model of functional extremum to derive the necessary conditions for the existence of functional extremum. This was done by introducing Lagrange multipliers to convert the problem of seismic active earth pressure into a function optimization problem with two undetermined function arguments. It can also be observed that when the point of action is at its lowest position and seismic activity is at a minimum, the slip surface can be observed to be a flat plane. However, as the point of action moves upwards, the slip surface gradually transforms into a logarithmic spiral, and the seismic active earth pressure increases in a nonlinear way. The highest value of seismic active earth pressure occurs when the point of action is at its uppermost position.

2.2.1.14 Onyelowe (2017)

Due to concerns over the collapse of buildings and highways in southeastern Nigeria, Onyelowe KC (2017) developed a mathematical technique to study the bearing capacity and normal stress distribution of soil foundations. The stability equations were derived using limit equilibrium conditions and represented as a minimization problem in calculus of variations. By minimizing the functional, the critical stress and load parameters that can lead to failure are determined, and the stability of the foundation depends on the soil's constitutive properties. The Meyerhoff and Hansen's superposition methods can be obtained using variational calculus, and the classical relationship between N_c and N_q is not influenced by the soil's constitutive law.

2.2.1.15 Li et al. (2018)

Li et al. (2018) developed a method for evaluating the impact of tension cracks on the stability of slopes in clays with linearly increasing undrained strength. It utilizes the limit equilibrium method with variational extremization to mathematically determine the normal stress distribution over the slip surface, which is then used to identify tension cracks in clays with zero tensile strength. The procedure also incorporates seismic effects through the pseudostatic approach. The minimum safety factor and maximum crack depth can be calculated using closed-form solutions, which are presented in charts for ease of use. The results indicate that neglecting tension cracks in traditional slope stability analyses can lead to overestimating slope safety, particularly in steep slopes under strong seismic conditions. The charts can also be used to determine the most vulnerable location of the tension crack, which can assist in designing reinforcement and remediation measures for slope stabilization.

2.2.1.16 Hua et al. (2022)

Hua et al. (2022) presented a variational method for analysing slope stability. The method was derived mathematically and physically, taking into account the entire system. The authors derived variational energy expressions for slopes by incorporating classical and engineering examples. They used the FISH language to implement calculation programmes in FLAC3D software and determined factors of safety by employing the variational method based on the strength reduction method (SRM).

2.2.1.17 Fengxi et al. (2023)

Fengxi demonstrated a theoretical analysis of soil slope stability using a combination of the variational method and the limit equilibrium method. They employed the limit equilibrium method to establish the basic equilibrium equation for the homogeneous slope stability problem and introduced the Lagrange multiplier to construct an auxiliary functional for the isoperimetric problem under constraints. They derived a set of first-order ordinary differential equations involving various unknowns, such as the potential sliding surface, normal stress of the sliding surface, internal force of the sliding body, safety factor, and Lagrange multiplier, by combining the Euler equation. By introducing auxiliary variables, the slope stability analysis was transformed into a two-point boundary value problem with fixed boundary conditions.

2.2.2 Upper bound rigid block method

The upper bound rigid block limit analysis is a computational technique that extends the analytical method developed by Drucker and Prager (1952). This method involves the use of simplified velocity fields, where rigid blocks slide along assumed failure surfaces. The collapse load is determined by equating the external work done to the internal power dissipation that occurs along the slip surface. The simplified upper bound technique yields highly accurate results for stability problems in homogeneous soil conditions if appropriate slip surfaces are selected. Some researchers carried out their work on stability problems utilizing the upper bound rigid block limit analysis, which is illustrated below:

2.2.2.1 Chen et al. (1969)

The upper bound theorem from the generalized theory of perfect plasticity was being utilized by Chen et al. (1969) to obtain comprehensive numerical solutions for determining the critical height of an embankment. In their study, it was assumed that a rotational discontinuity mechanism, specifically a logarithmic spiral, was at play. They also considered the slip surface passing at the toe of the embankment.

2.2.2.2 Chen and Giger (1971)

Chen and Giger (1971) extended the work of Chen et al. (1969) by assuming the slip surface passes beneath the toe of the embankment.

2.2.2.3 Chen (1975)

Chen (1975) explored various extensions of the fundamental concepts related to the plastic behavior of soils in the context of plane strain problems. Specifically, he examined the impact of factors such as heterogeneity, anisotropy, and broken slope gradients on the plasticity of soils.

2.2.2.4 Michalowski (1989)

Michalowski (1989) introduced a technique for the rigid block analysis of 3D slopes under local loading. The Michalowski (1989)’s method was different from previous approaches because it used multiple translating rigid blocks, up to 50, to model a rotational failure mechanism. A rigid-block toe, above-the-toe collapse mechanism was considered.

2.2.2.5 Michalowski (1995)

Michalowski (1995) presented a stability analysis of slopes using a translational mechanism of failure, based on rigid blocks similar to traditional slice methods. The analysis was based on the kinematic approach of limit analysis, but also ensured equilibrium of forces on all blocks. The proposed analysis provided a convenient way to include pore pressure effects and was implemented in the analysis of both translational and rotational slope collapse.

2.2.2.6 Donald and Chen (1997)

Donald and Chen (1997) proposed a method for stability analysis in soils and rocks, which was based on the upper bound theorem of classical plasticity. The method involved dividing the sliding mass into discrete blocks with linear interfaces and calculating the work done by external loads and body forces in comparison to the energy dissipated in shearing, resulting in the calculation of a safety or disturbance factor to predict failure mechanisms. The EMU (Energy method upper bound) computer program provided effective optimization routines for searching for the critical failure mechanism with the lowest factor of safety.

2.2.2.7 Farzaneh and Askari (2003)

Farzaneh and Askari (2003) analysed 3D homogeneous and non-homogeneous slopes by employing upper bound limit analysis. A rigid-block translational toe, above-the-toe or below-the-toe collapse mechanism is considered. The approach of Farzaneh and Askari (2003) is basically modification or extension of the methodology presented by Michalowski (1989).

2.2.2.8 Kumar and Samui (2006)

Kumar and Samui (2006) presented a method for computing stability numbers for two-layered soil slopes using upper bound limit analysis. The analysis assumed a rupture surface comprising logarithmic spiral arcs with a shared focus, and ensured that the collapse mechanism was kinematically admissible with regard to the rigid rotation of the soil mass. The method also took into account the impact of pore water pressure and horizontal earthquake body forces.

2.2.2.9 Farzaneh et al. (2008)

Farzaneh et al. (2008) used upper bound limit analysis to analyse 3D slopes which is convex in plan. A rigid-block translational collapse mechanism with energy dissipation over planar velocity discontinuities is considered. Farzaneh et al. (2008) concluded that convex slopes are more stable because they are not susceptible to surcharge loads.

2.2.2.10 Michalowski and Nadukuru (2013)

Michalowski and Nadukuru (2013) discussed the use of 3D slope stability analyses for assessing the safety of slopes, particularly in situations where the failure region's width was limited. They presented the kinematic approach of limit analysis, which utilized a rigid-rotation 3D mechanism that was modified to include below-toe failures commonly observed in gentle slopes. To account for the presence of pore water pressure in the slope, the scalar parameter r_u (coefficient of pore water pressure) was used as an approximation. Michalowski and Nadukuru (2013) found that the safety factor decreased with an increase in pore water pressure and an increased width of the slope.

2.2.2.11 Gao et al. (2014)

Gao et al. (2014) introduced a 3D rotational failure mechanism using the kinematic approach of limit analysis to examine the impact of water drawdown on the stability of 3D slopes. This led to the creation of stability charts that could conveniently estimate the safety factor of 3D slopes under different types of drawdown processes. Furthermore, it was discovered that when the width of a slope was sufficiently large (the ratio of the width to the height $B/H \geq 10.0$), the 3D effect could be disregarded, and the plane-strain analysis could be used to assess safety.

2.2.2.12 Zhao et al. (2016)

The study of Zhao et al. (2016) examined the stability of homogeneous slopes with cracks subjected to seismic loading by utilizing the upper bound limit analysis technique and the pseudo-static method. It presented a series of stability charts for slope inclinations and internal friction angles, which could be used to determine the critical slope height, critical crack depth, and the region affected by cracks for various crack scenarios such as cracks of known depth but unknown location, known location but unspecified depth, and cracks of unspecified depth and location.

2.2.2.13 Li and Yang (2019)

Li and Yang (2019) used a discretization method based on kinematical approach of upper bound theorem for estimating the factor of safety in soil slopes considering the nonlinearity of soil strength. The method included numerical simulation to determine the minor principal stress, calculating equivalent strength parameters based on the nonlinear failure criterion, discretization to generate a slip surface, and the upper bound theorem to derive the factor of safety.

2.2.2.14 Li et al. (2021)

Li et al. (2021) investigated the stability of soil slopes with multiple layers by employing a combination of the discrete technique and the upper bound approach. An algorithm was proposed to determine the coordinates of discrete points on the failure surface on the basis of geometrical relationships derived from the normality requirement. The sliding surface was generated from the starting point to the ending point, and the sliding block was sliced vertically. Internal energy dissipation and external work rates were calculated using integral equations. The strength reduction method and an optimisation algorithm were utilized to compute the safety factor.

2.2.2.15 Zuo et al. (2022)

Zuo et al. (2022) used a multicenter method to extend the log-spiral rotational mechanism to layered slopes. Instead of a single point of rotation, multiple rigid blocks were introduced, each with its own centre of rotation. Additional rupture surfaces between adjacent blocks were incorporated into the proposed mechanism. The proposed method, as well as other existing mechanisms based on limit equilibrium or limit analysis approaches, were employed to analyse a variety of examples.

2.3 LITERATURE REVIEW ON UNSATURATED MECHANICS AND APPLICATIONS

Unlike saturated soils, which are completely filled with water, unsaturated soils have some air occupying the void spaces between soil particles in addition to water. This coexistence of air and water in the soil pores introduces the hydro-mechanical behaviour of soils that differ from those of saturated soils. Significant progress has been made over the years in comprehending the behavior and modeling of unsaturated soils.

An important aspect of the advancements in unsaturated soil mechanics is the evolution of experimental techniques and instrumentation. Researchers have worked on refining laboratory testing methods to accurately measure soil-water characteristic curves (SWCC), which describe the relationship between soil suction and water content. Researchers have focused on enhancing laboratory testing methods to achieve precise measurements of soil-water characteristic curves (SWCC), which depict the correlation between soil suction (also known as soil matric suction) and water content. These SWCC curves are essential in assessing the hydro-mechanical characteristics of unsaturated soils. Numerous models based on SWCC have been developed, considering factors like suction, pore air pressure, and degree of saturation. The following section encompasses an in-depth literature review discussing the development and relevant applications of unsaturated soil mechanics in the context of geotechnical engineering.

2.3.1 Hilf (1956)

Hilf (1956) introduced the axis translation technique to increase pore water pressure above zero in laboratory tests, enabling the measurement of pore water pressure without disturbing soil structure. This technique became widely adopted for measuring or controlling suction in unsaturated soils. However, it had limitations, including preventing cavitation within pore spaces when pore water pressure changed from negative to positive values. Additionally, it was not suitable for measuring soil suction in in-situ devices. Understanding cavitation in unsaturated soils under in-situ stress conditions remained challenging with this technique.

2.3.2 Jennings and Burland (1962)

Jennings and Burland (1962) performed oedometer and all-round compression tests on partially and fully saturated soils, ranging from silty sand to silty clay. Their findings challenged the use of a single stress state variable approach in capturing the volume change behavior of partially saturated soils. They specifically questioned the suitability of Bishop's (1952) equation, based on a single effective stress variable, to predict wetting-induced collapse under constant total stress conditions. The study highlighted the limitations of the single stress state variable approach in accurately representing the behavior of partially saturated soils.

2.3.3 Fredlund et al. (1978)

Fredlund et al. (1978) compared three sets of triaxial data from Bishop et al. (1960) and MIT (1963) using two independent sets of stress variables. They proposed two sets of equations for shear strength: the first equation assumed a planar and linear surface of failure, while the second equation considered (σ-u_a) and (u_a-u_w) as prime variables. After analyzing the data, they found that the second equation was the most practical and useful for engineering applications.

2.3.4 van Genuchten (1980)

van Genuchten introduced a novel equation for the soil-water content-pressure head curve, providing a simple approach. When integrated into the predictive conductivity models by Burdine (1953) or Mualem (1976), closed-form analytical expressions for the relative hydraulic conductivity, Kr, could be derived. The study compared these expressions, based on Mualem's theory, with observed hydraulic conductivity data from five soils with varying hydraulic properties, and found accurate predictions of unsaturated hydraulic conductivity. The research underscored the importance of accurately describing the soil-water retention curve at low water contents for effective estimation of unsaturated hydraulic conductivity

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2.3.5 Fredlund and Xing (1994)

Fredlund and Xing proposed a comprehensive equation to estimate parameters characterizing unsaturated soil behavior, particularly focusing on creating a soil-water characteristic curve. They used a nonlinear least-squares computer program to fit the equation to experimental data from existing literature. The equation was based on the assumption that the soil-water characteristic curve's shape is influenced by the soil's pore-size distribution, which governs the desaturation process. Their equation exhibited excellent performance across a wide range of suction values (0 to 106 kPa) for various soil types, including sand, silt, and clay. This equation provides a valuable tool for estimating critical parameters in unsaturated soil behavior based on experimental data.

2.3.6 Vanapalli and Fredlund (1996)

Vanapalli and Fredlund (1996) conducted an extensive study investigating the correlation between shear strength and the soil-water characteristic curve of unsaturated soil, with a specific focus on matric suction. They developed an analytical model to predict shear strength by considering soil suction, along with parameters related to the soil-water characteristic curve and saturated shear strength. To validate the model, they conducted experiments on glacial till specimens under various conditions, including different water contents, densities, net normal stresses, and matric suctions. The results demonstrated a strong relationship between the predicted shear strength values and the measured values. The model, based on a theory initially proposed by Fredlund et al. (1978), offered two different approaches to estimate shear strength using effective shear strength parameters and the soil-water characteristic curve.

2.3.7 Tekinsoy et al. (2004)

Tekinsoy et al. (2004) introduced an empirical-analytical function to estimate the suction strength of unsaturated soils, utilizing the soil-water characteristic curve (SWCC) and the effective angle of shearing resistance. Their model relied on a single value from the SWCC, known as the air entry value (α), to determine the variation of suction strength with matric suction. The logarithmic model proposed in the study offered a simple and convenient alternative approach. Testing the model with published data showed that the calculated suction strengths aligned well with the measured data, demonstrating the effectiveness of the formula for estimating suction strength in engineering applications. The study concluded that the proposed model was particularly suitable for fine-grained soils.

2.3.8 Sheng et al. (2009)

Sheng et al. (2009) conducted a study evaluating various shear strength equations' performance in predicting the mechanical behavior of unsaturated soils. They observed that the shear strength of soil showed a nonlinear relationship with suction, increasing with the effective angle of internal friction at low suction values and gradually decreasing as residual suction conditions were approached. The study emphasized the significance of considering material parameters and specific data sets when selecting and applying shear strength equations to predict the mechanical behavior of unsaturated soils. These findings were valuable for researchers and practitioners in geotechnical engineering and soil mechanics, especially for designing foundations, embankments, and other geotechnical structures subjected to unsaturated soil conditions.

2.3.9 Kim and Borden (2011)

Kim and Borden (2011) conducted a study comparing three empirical procedures' ability to predict shear strength in 15 soils within the 0-200 kPa range. The results showed that the prediction methods provided good comparisons for low-plasticity clays but tended to overestimate shear strengths for sandy soils. Additionally, in the higher net normal stress range, the predicted shear strengths highly underestimated the measured values compared to the lower net normal stress range. The study also found that shear strengths predicted using soil-water characteristic curves (SWCCs) performed at different net normal stresses were significantly higher than those predicted using a single SWCC performed at zero or nominal net normal stress. For soils with $n > 2$ (where n is the slope of the SWCC), the predicted shear strengths were highly underestimated for fine-grained soils (clay or silt) but slightly overestimated for sandy soils.

2.3.10 Fredlund (2018)

Fredlund discussed various methods used to address hysteresis complications in unsaturated soil mechanics. One approach involved measuring both drying and wetting bounding curves in the laboratory. Alternatively, only measuring the drying curve and using it to estimate unsaturated soil property functions was another option, though it could introduce bias. Another method involved estimating the wetting soil-water characteristic curve from the drying SWCC and computing a "median" SWCC for specific unsaturated soil problems. Crucially, these estimation procedures needed to be combined with fundamental unsaturated soil property functions measured in the laboratory, such as the drying gravimetric water content SWCC and the shrinkage curve.

2.3.11 Vahedifard et al. (2016)

Vahedifard et al. (2016) introduced an analytical approach for assessing the stability of unsaturated homogeneous slopes under steady vertical flow using effective stress limit-equilibrium (LE) analysis. The method employed a suction stress-based representation of effective stress, which incorporated a log spiral failure surface and two additional hydromechanical parameters which are related to inverse of air entry pressure and pore size distribution to describe seepage and effective stress variations in unsaturated soils. They investigated how infiltration and evaporation affected the slope stability of four hypothetical soil types. Notably, the study highlighted the significant role of apparent cohesion resulting from suction stress in slope stability. Furthermore, different seepage rates were found to have significant impacts on the stability of clayey slopes, with a diminishing influence observed in silty, loess, and sandy slopes.

2.3.12 Sun et al. (2019)

Sun et al. (2019) presented an analytical framework to assess the stability of unsaturated soil slopes with tension cracks, focusing on one-dimensional steady-infiltration conditions and utilizing the limit-analysis method. The study took into account the impact of suction stress, effective unit weight of soil, and tension cracks by formulating a boundary-value problem based on the log-spiral failure mechanism. The research extensively explored the effects of various factors, including suction, Poisson's ratio, steady infiltration, and water presence in tension cracks, on slope stability.

2.4 SUMMARY

The present chapter provides a comprehensive literature review on variational method, upper bound rigid block method, and unsaturated soil mechanics. The first section showcases the application of variational method and upper bound rigid block method in various stability problems. The literature review on these methods highlights the significance of these methods in geotechnical engineering and stability analysis. Despite advancements in geotechnical engineering, the vastness of geotechnical structures and the diverse properties of geo-materials give rise to multiple unresolved problems. The second section is dedicated to exploring the role of unsaturated mechanics in geotechnical engineering through an extensive literature review. The evolution of unsaturated mechanics in geotechnical engineering has led realize the soil behavior more realistically. However, based on unsaturated soil mechanics, proper design charts concerning the slope stability and earth-pressure behind the retaiing walls are not readily available. In this context, the subsequent chapters may be filled up the research gap.