Chapter 7
Conclusions and Future Remarks

The thesis is focused on fractal interpolation and finding the dimensional results of graphs of fractal functions. This chapter concludes the thesis and presents some possible future work in the direction of work done in this thesis.

7.1 Conclusion

This section serves as the conclusion of the entire thesis, summarizing the important findings and concluding remarks from the overall investigation.

Chapter 1 has introduced the fundamental concepts and literature related to the work presented in this thesis.

Chapter 2 has presented the examination of the fractal dimension of vector-valued functions. Specifically, the fractal dimension of the graph of the Katugampola fractional integral of a vector-valued continuous function with bounded variation, which is defined on a closed and bounded interval of the real numbers, is explored.

Chapter 3 has proposed the concept of multivariate fractal interpolation functions, which are used to analyze a given set of data points. The chapter has also explored the existence of the α-fractal function, which corresponds to a multivariate continuous function defined on the domain $[0,1]^N$, where N is a natural number.

Additionally, the behavior of the α-fractal function when restricted to the coordinate axis is studied. Furthermore, the box dimension and the Hausdorff dimension of the graph of the multivariate α-fractal function and its restriction are investigated.

Chapter 4 has hinted at the construction of dimension-preserving approximation of multivariate continuous functions. Moreover, the existence of a one-sided approximation of multivariate functions using fractal functions is proved. Also, the existence of the Schauder basis consisting of multivariate fractal functions for the space of all real-valued continuous functions defined on $[0, 1]^N$ is established.

Chapter 5 has initiated the study of α-fractal function and fractal approximation for set-valued functions. Also, the study of the fractal dimension of the graph of some special class of functions is given in this chapter.

Chapter 6 has discussed the properties of invariant measure generated by the system of IFS constructed by Barnsley [9]. Furthermore, the fractal transformation between two FIFs that have identical interpolation data is given. Towards the end of the chapter, specific function spaces were introduced, along with the criteria for determining whether a FIF can be classified as a member of these spaces.

7.2 Some Future Directions

In spite of continuous research in the field of fractal theory, significant issues exist in this area. Fractal interpolation techniques have many real-world applications in different areas, see, for instance, [75, 95, 97], etc. The study of fractal interpolation and fractal dimension are burgeoning topics due to their applicability in different areas.

Based on the research presented in this thesis, there are various opportunities for further expansion. The following are some potential future problems one can address.

- In Chapter 2, the research focuses on the fractal dimension of vector-valued maps. The study suggests that instead of getting the upper bound of the fractal dimension of the vector-valued maps, one can analyze the bound for the coordinates maps if the vector-valued map is Hölder continuous (Theorem 2.28). In view of Theorem 2.28, in the future, it would be interesting to explore the bound for the dimension of the graph of ${}_a^{\rho}\mathfrak{J}^{\mu}\boldsymbol{f}$.

- In [76, 77, 78], authors have used ϕ-contractions to produce the system of IFS whose attractor is a FIF corresponding to the given data set, unlike the case of Barnsley [9], where he has used Banach contraction. A plethora of results have been given to study fractal dimension and fractional calculus regarding the FIF constructed by Barnsley. In the future, one may try to work on the fractal dimensional and fractional calculus of these generalized FIFs. Also, one can study the properties of invariant measures corresponding to these generalized FIFs.

- The concept of a set-valued map is extensively useful in game theory, control theory, optimization, etc. Many theories related to the classical integral of set-valued map is introduced in the literature (see for instance [5, 62]). One can extend the concept of fractional integral of set-valued map and investigate the dimension of the graph of fractional integrals of set-valued maps.

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