

**STUDY OF STABILITY AND SYNCHRONIZATION OF
SOME PROBLEMS RELATED TO NEURAL
NETWORKS**



*The thesis submitted in partial fulfilment
for the Award of Degree
DOCTOR OF PHILOSOPHY*

by

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Chapter 6

Summary of the thesis and future work

6.1 Summary

This thesis deals with some problems with the stability and synchronization of different neural networks. The systems which are to be synchronized are called master and slave systems. The synchronization analysis is obtained through the stability analysis of the error system obtained by subtracting the state vectors of the master system from those of the slave system. In some cases, the scaling factor is used to obtain the error systems to achieve synchronization called projective synchronization. It all depends on the kind of stability we are analyzing. One more interesting thing is the effect of time delays in the actual implementation of the neural networks. The neural networks with time delays are the main focus of the present thesis. The first chapter gives a glimpse into the basic neural network structure and its working. To

ease the things, some neural network models are elaborated. The chapter concludes with an insight into the methods and lemmas employed in the thesis.

Chapter 2 focuses on the fixed-time synchronization of neural networks. Fixed-time synchronization is the extension of the finite-time synchronization. The trajectory of the solution curve reaches the equilibrium point in finite time together with the advantage that this time is independent of the initial condition. Here, the quaternion-valued neural networks have been explored. The quaternion-valued neural networks are separated into four real-valued neural networks. Then using the lemmas defined in the first Chapter, sufficient conditions for the synchronization of neural networks are obtained. A comparison of the settling time using two different lemmas is also shown analytically as well as numerically. Finally, a numerical result authenticates the findings.

Chapter 3 deals with the complex-valued neural networks, where the key challenge was the effects of heterogeneous impulses on the synchronization of the neural networks. The neural networks considered in this chapter are categorized into two parts. One is changing with time, and other is changing with the nodes. These kinds of impulses are known as heterogeneous impulses. The concepts of average impulsive gain and average impulsive interval are also used. The sufficient conditions are obtained by using the Halanay inequality together with the concept of Matrix measure.

Chapter 4 is about the global exponential stability of neural networks. The neural network considered is the octonion-valued neural network. In exponential stability, the trajectories of the neural network approaches the equilibrium point as time goes to infinite with the exponential rate. The octonion-valued neural networks fall under the category of hyper-complex neural networks. These are neither commutative nor associative. The activation functions are assumed to satisfy the Lipschitz condition.

The sufficient conditions for the exponential stability of neural networks are obtained using the extended Halanay inequality. Finally, numerical examples are given to validate the results.

Chapter 5 takes into account the Cohen-Grossberg neural network. The non-reduced order method discusses two kinds of lag synchronization of the said neural network. Two kinds of Lyapunov functional, together with the novel controller construction, helped achieve the desired synchronization. The first one is the design of the controller, which is the feedback controller, and the other one is the adaptive controller. The results are validated by the numerical example given at the end of the chapter.

6.2 Future work

Our main works in this thesis are mainly concerned with the stability and synchronization of neural networks. Two different kinds of stability have been explored. The second chapter is concerned with the fixed-time synchronization of the neural networks. However, the settling time considered is independent of initial conditions. But in the actual implementation of neural networks, finding the exact initial conditions is not always possible. To overcome this, we have a novel, pre-defined stability. Here, the settling time can be tuned in advance. Therefore, there is enough scope to explore this with different delays and impulses, which have been briefly discussed in the third chapter. Also, the predefined time stability of the neural network models of chapters 4 and 5 are yet to be explored. Based on the above discussion, the following future works can be done.

- . Predefined and input to state stability of neural networks.
- . Effects of impulses on the predefined time synchronization of the Cohen-Grossberg neural networks.

- . Analysis of global dissipativity with hybrid impulses and non-differentiable delays.
- . To look into the dynamical properties of the octonion-valued neural networks because they have been explored very less.
