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Appendix A. Some expressions given in Chapter 2

Expressions of different notations of the section 2.3 are given as

$$k(x,t) = \lim_{y \to 0^-} \int_0^\infty \left(\frac{2\rho \, \exp(n_1 y)}{\sqrt{\delta^2 + 4\rho^2}} - 1 \right) [\sin(\rho(t+x)) + \sin(\rho(t-x))] \, d\rho, \qquad \text{(A.0.1)}$$

where n_1 is given in (2.26).

$$k_{11}(\xi_1, \eta_1) = ak(a\xi_1, a\eta_1), \qquad \text{(A.0.2)}$$

$$k_{12}(\xi_1, \eta_2) = \frac{c-b}{2} k(a\xi_1, \frac{(c-b)\eta_2 + c + b}{2}), \qquad \text{(A.0.3)}$$

$$k_{21}(\xi_2, \eta_1) = ak(\frac{(c-b)\xi_2 + c + b}{2}, a\eta_1), \qquad \text{(A.0.4)}$$

$$k_{22}(\xi_2, \eta_2) = \frac{c-b}{2} k(\frac{(c-b)\xi_2 + c + b}{2}, \frac{(c-b)\eta_2 + c + b}{2}), \qquad \text{(A.0.5)}$$

$$g_{12}(\xi_1) = -\frac{2a\xi_1 + c + b}{c-b}, \qquad \text{(A.0.6)}$$

$$g_{22}(\xi_2) = -\frac{(c-b)\xi_2 + 2c + 2b}{c-b}, \qquad \text{(A.0.7)}$$

$$z_{12}(\xi_1) = \frac{2a\xi_1 - c - b}{c-b}, \qquad \text{(A.0.8)}$$

$$z_{21}(\xi_2) = \frac{(c-b)\xi_2 + c + b}{2a}, \qquad \text{(A.0.9)}$$

$$\frac{1}{\pi}\int_{-1}^{1}\frac{T_n(t)}{(t-x)\sqrt{1-t^2}}dt = \begin{cases} 0 & n=0,\ 0<x<1, \\ U_{n-1}(x), & n\geq 1,\ 0<x<1, \\ -\frac{(x-\sqrt{x^2-1})^n}{\sqrt{x^2-1}}, & x>1, \end{cases} \tag{A.0.10}$$

where T_n and U_n are the Chebyshev polynomials of first and second kinds, respectively. Expressions of different notations of the section (2.4) are given as

$$s_j = \frac{(k-1)m_j^2+\beta(k-1)m_j-\rho^2(k+1)}{\rho(2m_j+\beta(k-1))},\ j=1,...,4 \tag{A.0.11}$$

$$d_1 = (\gamma+n_2)^4+2\beta(\gamma+n_2)^3+(\beta^2-2\rho^2)(\gamma+n_2)^2-2\beta\rho^2(\gamma+n_2)+\rho^4+\rho^2\beta^2c^2, \tag{A.0.12}$$

$$d_2 = (\gamma+n_1)^4+2\beta(\gamma+n_1)^3+(\beta^2-2\rho^2)(\gamma+n_1)^2-2\beta\rho^2(\gamma+n_1)+\rho^4+\rho^2\beta^2c^2, \tag{A.0.13}$$

$$\zeta_1 = -\frac{4\rho A_2}{k^2-1}[(k+1)(\gamma+n_2)^2+\beta(k+1)(\gamma+n_2)-\rho^2(k-1)-(\beta+\gamma+n_2) \times (2(\gamma+n_2)+\beta(k-1))], \tag{A.0.14}$$

$$\zeta_2 = \frac{4A_2}{k^2-1}[\rho^2(2(\gamma+n_2)+\beta(3-k))+(\beta+\gamma+n_2)((k-1)(\gamma+n_2)^2+\beta(k-1) \times (\gamma+n_2)-\rho^2(k-1))], \tag{A.0.15}$$

$$\zeta_3 = -\frac{4\rho n_2 A_2}{(k^2-1)n_1}[(k+1)(\gamma+n_1)^2+\beta(k+1)(\gamma+n_1)-\rho^2(k-1)-(\beta+\gamma+n_1) \times (2(\gamma+n_1)+\beta(k-1))], \tag{A.0.16}$$

$$\zeta_4 = \frac{4n_2 A_2}{(k^2-1)n_1}[\rho^2(2(\gamma+n_1)+\beta(3-k))+(\beta+\gamma+n_1)((k-1)(\gamma+n_1)^2+\beta(k-1) \times (\gamma+n_1)-\rho^2(k-1))], \tag{A.0.17}$$

$$p_k = m_k s_k(k+1)+\rho(3-k), \tag{A.0.18}$$

$$q_k = m_k-\rho s_k, \tag{A.0.19}$$

$$e_1 = \frac{(1-k^*)Q_0\alpha_0}{k_0}\left[\frac{\rho\zeta_2-\zeta_1(\gamma+n_2)}{d_1}+\frac{\zeta_3(\gamma+n_1)-\rho\zeta_4}{d_2}\right], \tag{A.0.20}$$

$$e_2 = \frac{(1-k^*)Q_0\alpha_0}{k_0}\left[\frac{(3-k)\rho\zeta_3 + (1+k)(\gamma+n_1)\zeta_4}{d_2} - \frac{(3-k)\rho\zeta_1 + (1+k)(\gamma+n_2)\zeta_2}{d_1}\right], \tag{A.0.21}$$

$$f_1 = \frac{2}{\pi\rho}\left[\int_0^a \psi_u \cos(\rho t)dt + \int_b^c \psi_u \cos(\rho t)dt\right], \tag{A.0.22}$$

$$f_2 = -\frac{2}{\pi\rho}\left[\int_0^a \psi_v \sin(\rho t)dt + \int_b^c \psi_v \sin(\rho t)dt\right], \tag{A.0.23}$$

$$f_{11} = \frac{(1-k^*)Q_0\alpha_0}{k_0}\left[\frac{\zeta_3}{d_2} - \frac{\zeta_1}{d_1}\right], \tag{A.0.24}$$

$$f_{22} = \frac{(1-k^*)Q_0\alpha_0}{k_0}\left[\frac{\zeta_4}{d_2} - \frac{\zeta_2}{d_1}\right], \tag{A.0.25}$$

where $n_i(i=1,2)$, $m_i(i=1,...,4)$ are given in (2.26) and (2.44), respectively.

$$k_{u1}(x,t) = \lim_{y\to 0^-}\int_0^{+\infty}\left(\frac{1+k}{4}N_{11}(y,\rho) - 1\right)\left[\sin(\rho(t-x)) - \sin(\rho(t+x))\right]d\rho, \tag{A.0.26}$$

$$k_{u2}(x,t) = \lim_{y\to 0^-}\int_0^{+\infty}\left(\frac{1+k}{4}M_{12}(y,\rho)\right)\left[\cos(\rho(t-x)) - \cos(\rho(t+x))\right]d\rho, \tag{A.0.27}$$

$$k_{\nu 1}(x,t) = \lim_{y\to 0^-}\int_0^{+\infty}\left(\frac{1+k}{4(k-1)}M_{21}(y,\rho)\right)\left[\cos(\rho(t+x)) + \cos(\rho(t-x))\right]d\rho, \tag{A.0.28}$$

$$k_{\nu 2}(x,t) = \lim_{y\to 0^-}\int_0^{+\infty}\left(\frac{1+k}{4(k-1)}N_{22}(y,\rho) - 1\right)\left[\sin(\rho(t+x)) + \sin(\rho(t-x))\right]d\rho, \tag{A.0.29}$$

$$N_{11}(y,\rho) = \frac{2}{\rho D}\left[-q_1 D_{33}\exp(m_1 y) + q_3 D_{34}\exp(m_3 y)\right], \tag{A.0.30}$$

$$M_{12}(y,\rho) = \frac{2}{\rho D}\left[q_1 D_{43}\exp(m_1 y) - q_3 D_{44}\exp(m_3 y)\right], \tag{A.0.31}$$

$$M_{21}(y,\rho) = \frac{2}{\rho D}\left[p_1 D_{33}\exp(m_1 y) - p_3 D_{34}\exp(m_3 y)\right], \tag{A.0.32}$$

$$N_{22}(y,\rho) = \frac{2}{\rho D}\left[p_1 D_{43}\exp(m_1 y) - p_3 D_{44}\exp(m_3 y)\right], \tag{A.0.33}$$

$$g_1 = \frac{-q_1 D_{13} + q_3 D_{14}}{D}, \tag{A.0.34}$$

$$g_2 = \frac{q_1 D_{23} - q_3 D_{24}}{D}, \tag{A.0.35}$$

$$g_3 = \frac{-q_1 D_{33} + q_3 D_{34}}{D}, \tag{A.0.36}$$

$$g_4 = \frac{q_1 D_{43} - q_3 D_{44}}{D}, \tag{A.0.37}$$

$$g_5 = \frac{(1-k^*)Q_0\alpha_0}{k_0}\left[\frac{\rho\zeta_4 - (\gamma + n_1)\zeta_3}{d_2}\right], \tag{A.0.38}$$

$$h_1 = \frac{-p_1 D_{13} + p_3 D_{14}}{D}, \tag{A.0.39}$$

$$h_2 = \frac{p_1 D_{23} - p_3 D_{24}}{D}, \tag{A.0.40}$$

$$h_3 = \frac{-p_1 D_{33} + p_3 D_{34}}{D}, \tag{A.0.41}$$

$$h_4 = \frac{p_1 D_{43} - p_3 D_{44}}{D}, \tag{A.0.42}$$

$$h_5 = \frac{(1-k^*)Q_0\alpha_0}{k_0}\left[4A_3(\rho) - \frac{(\gamma + n_1)(1+k)\zeta_4 + (3-k)\rho\zeta_3}{d_2}\right], \tag{A.0.43}$$

where n_i, m_i and $A_3(\rho)$ are given in (2.26), (2.44), (2.31) and (2.32) respectively. D is the determinant and D_{ij} 's $(i, j = 1, ..., 4)$ are the sub-determinants obtained by eliminating i^{th} row and j^{th} coloumn of the matrix in (2.50).

$$k_{u11}(\xi_1, \eta_1) = ak_{u1}(a\xi_1, a\eta_1), \tag{A.0.44}$$

$$k_{u12}(\xi_1, \eta_2) = \frac{c-b}{2}k_{u1}(a\xi_1, \frac{(c-b)\eta_2 + c + b}{2}), \tag{A.0.45}$$

$$k_{u21}(\xi_2, \eta_1) = ak_{u1}(\frac{(c-b)\xi_2 + c + b}{2}, a\eta_1), \tag{A.0.46}$$

$$k_{u22}(\xi_2, \eta_2) = \frac{c-b}{2}k_{u1}(\frac{(c-b)\xi_2 + c + b}{2}, \frac{(c-b)\eta_2 + c + b}{2}), \tag{A.0.47}$$

$$k_{u31}(\xi_1, \eta_1) = ak_{u2}(a\xi_1, a\eta_1), \tag{A.0.48}$$

$$k_{u32}(\xi_1, \eta_2) = \frac{c-b}{2}k_{u2}(a\xi_1, \frac{(c-b)\eta_2 + c + b}{2}), \tag{A.0.49}$$

$$k_{u41}(\xi_2, \eta_1) = ak_{u2}(\frac{(c-b)\xi_2 + c + b}{2}, a\eta_1), \tag{A.0.50}$$

$$k_{u42}(\xi_2, \eta_2) = \frac{c-b}{2}k_{u2}(\frac{(c-b)\xi_2 + c + b}{2}, \frac{(c-b)\eta_2 + c + b}{2}), \tag{A.0.51}$$

$$k_{\nu 11}(\xi_1, \eta_1) = ak_{\nu 1}(a\xi_1, a\eta_1), \tag{A.0.52}$$

$$k_{\nu 12}(\xi_1, \eta_2) = \frac{c-b}{2} k_{\nu 1}(a\xi_1, \frac{(c-b)\eta_2 + c + b}{2}), \tag{A.0.53}$$

$$k_{\nu 21}(\xi_2, \eta_1) = a k_{\nu 1}(\frac{(c-b)\xi_2 + c + b}{2}, a\eta_1), \tag{A.0.54}$$

$$k_{\nu 22}(\xi_2, \eta_2) = \frac{c-b}{2} k_{\nu 1}(\frac{(c-b)\xi_2 + c + b}{2}, \frac{(c-b)\eta_2 + c + b}{2}), \tag{A.0.55}$$

$$k_{\nu 31}(\xi_1, \eta_1) = a k_{\nu 2}(a\xi_1, a\eta_1), \tag{A.0.56}$$

$$k_{\nu 32}(\xi_1, \eta_2) = \frac{c-b}{2} k_{\nu 2}(a\xi_1, \frac{(c-b)\eta_2 + c + b}{2}), \tag{A.0.57}$$

$$k_{\nu 41}(\xi_2, \eta_1) = a k_{\nu 2}(\frac{(c-b)\xi_2 + c + b}{2}, a\eta_1), \tag{A.0.58}$$

$$k_{\nu 42}(\xi_2, \eta_2) = \frac{c-b}{2} k_{\nu 2}(\frac{(c-b)\xi_2 + c + b}{2}, \frac{(c-b)\eta_2 + c + b}{2}), \tag{A.0.59}$$

$$w_1(\xi_1) = w_1(a\xi_1), \tag{A.0.60}$$

$$w_1(\xi_2) = w_1(\frac{(c-b)\xi_2 + c + b}{2}), \tag{A.0.61}$$

$$w_2(\xi_1) = w_2(a\xi_1), \tag{A.0.62}$$

$$w_2(\xi_2) = w_2(\frac{(c-b)\xi_2 + c + b}{2}). \tag{A.0.63}$$

Appendix B. Some expressions given in Chapter 3

$$\kappa_i(\tau,p) = (\gamma+\beta)c_i(\tau,p) + (\gamma-\beta)d_i(\tau,p), \qquad i = 1,2, \tag{B.0.1}$$

$$R(s,x,p) = \int_0^\infty \left[\frac{\kappa_1(\tau,p)}{\tau} - \frac{1}{2}\right](\sin\tau(s-x) + \sin\tau(s+x))\,d\tau, \tag{B.0.2}$$

$$T_t(x) = -\frac{1}{\mu_1\pi}\int_0^\infty \sigma_0(s)\left[\int_0^\infty \frac{\kappa_2(\tau,p)}{\alpha}(\cos\tau(s-x) + \cos\tau(s+x))\,d\tau\right]ds, \tag{B.0.3}$$

$$T_c(x) = -\frac{1}{\mu_2}\sigma_0(x), \tag{B.0.4}$$

$$T_1(x) = \begin{cases} -\frac{\sigma_0}{\mu_1\pi}\int_0^\infty \frac{\kappa_2(\tau,p)}{\alpha\tau}(\sin\tau(L-x) & \\ +\sin\tau(L+x))d\tau, & \text{when impact load acts on upper material surface,} \\ -\frac{1}{\mu_2}\sigma_0 H(L-|x|), & \text{when impact load acts on cracked surface,} \end{cases} \tag{B.0.5}$$

$$R_{11}(\eta_1,\xi_1,p) = aR(a\eta_1, a\xi_1, p), \tag{B.0.6}$$

$$R_{12}(\eta_2,\xi_1,p) = (c-b)R(\frac{(c-b)\eta_2+c+b}{2}, a\xi_1, p), \tag{B.0.7}$$

$$R_{21}(\eta_1,\xi_2,p) = aR(a\eta_1, \frac{(c-b)\xi_2+c+b}{2}, p), \tag{B.0.8}$$

$$R_{22}(\eta_2,\xi_2,p) = (c-b)R(\frac{(c-b)\eta_2+c+b}{2}, \frac{(c-b)\xi_2+c+b}{2}, p), \tag{B.0.9}$$

$$z_{12}(\xi_1) = \frac{2a\xi_1-(b+c)}{c-b}, \tag{B.0.10}$$

$$z_{21}(\xi_2) = \frac{(c-b)\xi_2+c+b}{2a}, \tag{B.0.11}$$

$$g_{12}(\xi_1) = -\left[\frac{2a\xi_1+b+c}{c-b}\right], \tag{B.0.12}$$

$$g_{22}(\xi_2) = -\left[\frac{(c-b)\xi_2 + 2c + 2b}{c-b}\right], \tag{B.0.13}$$

$$T_{11}(\xi_1) = T_1(a\xi_1), \tag{B.0.14}$$

$$T_{12}(\xi_2) = T_1(\frac{(c-b)\xi_2 + c + b}{2}), \tag{B.0.15}$$

$$\xi_{ij} = \cos\left[\frac{(2j-1)\pi}{2n_i}\right], \qquad \eta_{ik} = \cos\left[\frac{k\pi}{n_i}\right], \qquad \zeta_k = \begin{cases} 0.5, & k = 0, n_i, \\ 1, & k = 1, ..., n_i - 1, \end{cases}$$

$$j = 1, 2, ..., n_i, \qquad k = 0, ..., n_i, \qquad i = 1, 2. \tag{B.0.16}$$

Appendix C. Some expressions given in Chapter 4.1

$$F^{(2)}_{tnj}(s) = is\cos(\theta) + \lambda^{(2)}_{nj}(s)\sin(\theta), \tag{C.0.1}$$

$$I_{tnj}(s,\alpha,x) = \int_{-\infty}^{\infty} e^{\lambda^{(2)}_{nj}(s)y_0 - isx_0 + i\alpha y} dy, \tag{C.0.2}$$

$$F^{(1)}_{tnj}(\alpha) = i\alpha\cos(\theta) + \lambda^{(1)}_{nj}(\alpha)\sin(\theta), \tag{C.0.3}$$

$$N_n(s,x_0) = W^{(2)}_{n1}(s)\lambda^{(2)}_{n1}(s)e^{-isx_0} - \lim_{y_0\to 0^+}\sum_{j=1}^{2}\frac{1}{2\pi}\int_{-\infty}^{\infty} G_{2nj}(\alpha,s)F^{(1)}_{tnj}(\alpha)e^{\lambda^{(1)}_{nj}(\alpha)x - i\alpha y}d\alpha, \tag{C.0.4}$$

$$S_n(x_0) = \int_{-\infty}^{\infty}\lim_{y_0\to 0^+}\left[\sum_{j=1}^{2} F^{(1)}_{tnj}(\alpha)G^{(1)}_{1nj}(\alpha)e^{\lambda^{(1)}_{nj}(\alpha)x - i\alpha y}\right]d\alpha, \tag{C.0.5}$$

where $n = 1,2; j = 1,2.$

$$E^{(1)}_{nj}(\alpha) = \frac{\tau^{(1)}_{nj}(\alpha)\left(\tau^{(1)}_{nj}(\alpha) + \beta_n\right) - f_1\alpha^2}{i\alpha\left((f_2+1)\tau^{(1)}_{nj}(\alpha) + \beta_n\right)},\ f_1 = \frac{1+k}{k-1},\ f_2 = \frac{3-k}{k-1},\ n = 1,2; j = 1,2,3,4, \tag{C.0.6}$$

$$\begin{aligned}\eta^{(1)}_{nj}(\alpha) = &-\alpha^2\left[2\lambda^{(1)}_{nj}(\alpha) + \beta_n(3-k)\right]\\ &-\left[\gamma_n + \beta_n + \lambda^{(1)}_{nj}(\alpha)\right]\left[(k-1)\lambda^{(1)}_{nj}(\alpha)^2 + \beta_n(k-1)\lambda^{(1)}_{nj}(\alpha) - (1+k)\alpha^2\right],\end{aligned} \tag{C.0.7}$$

$$\zeta_{nj}^{(1)}(\alpha) = i\alpha\left[(1+k)\lambda_{nj}^{(1)}(\alpha)\left(\lambda_{nj}^{(1)}(\alpha)+\beta_n\right) - \alpha^2(k-1)\right]$$
$$- i\alpha\left[\gamma_n + \beta_n + \lambda_{nj}^{(1)}(\alpha)\right]\left[2\lambda_{nj}^{(1)}(\alpha) + \beta_n(k-1)\right], \tag{C.0.8}$$
$$d_{nj}^{(1)}(\alpha) = (1-k^2)\left[\left(\lambda_{nj}^{(1)}(\alpha)^2 + \lambda_{nj}^{(1)}(\alpha)\beta_n - \alpha^2\right)^2 + f_0^2\alpha^2\beta_n^2\right], \tag{C.0.9}$$

where $n = 1,2; j = 1,2$.

$$E_{nj}^{(2)}(s) = \frac{\tau_{nj}^{(2)}(s)\left(\tau_{nj}^{(2)}(s)+\beta_{n2}\right)f_1 - s\,(s+i\beta_{n1})}{i\left[\tau_{nj}^{(2)}(s)\,(s+i\beta_{n1}) + f_2(\tau_{nj}^{(2)}(s)+\beta_{n2})s\right]}, \qquad n=1,2; j=1,2,3,4, \tag{C.0.10}$$

$$\eta_{nj}^{(2)}(s) = \left[\gamma_{n2}+\beta_{n2}+\lambda_{nj}^{(2)}(s)\right]\left[((3-k)\beta_{n1}-2is)\,\lambda_{nj}^{(2)}(s) - is(k-1)\beta_{n2}\right]$$
$$- \left[\gamma_{n1}+\beta_{n1}-is\right]\left[(1+k)\lambda_{nj}^{(2)}(s)(\lambda_{nj}^{(2)}(s)+\beta_{n2}) - (k-1)s(s+i\beta_{n1})\right], \tag{C.0.11}$$

$$\zeta_{nj}^{(2)}(s) = \left[\gamma_{n1}+\beta_{n1}-is\right]\left[(k-1)\beta_{n1}\lambda_{nj}^{(2)}(s) - is(3-k)\beta_{n2} - 2is\lambda_{nj}^{(2)}(s)\right]$$
$$- \left[\lambda_{nj}^{(2)}(s)+\gamma_{n2}+\beta_{n2}\right]\left[(k-1)\lambda_{nj}^{(2)}(s)(\lambda_{nj}^{(2)}(s)+\beta_{n2}) - s(1+k)(s+i\beta_{n1})\right], \tag{C.0.12}$$

$$d_{nj}^{(2)}(s) = (1-k^2)f_0^2\left[s\beta_{n2} - i\lambda_{nj}^{(2)}(s)\beta_{n1}\right]^2 + (1-k^2)\left[\lambda_{nj}^{(2)}(s)\left(\lambda_{nj}^{(2)}(s)+\beta_{n2}\right) - s\,(s+i\beta_{n1})\right]^2,$$

where $n = 1,2; j = 1,2$.

$$B_{nj}^{(1)}(\alpha) = f_1E_{nj}^{(1)}(\alpha)\tau_{nj}^{(1)}(\alpha) - f_2i\alpha, \quad B_{nj}^{(2)}(s) = f_2\tau_{nj}^{(2)}(s) - isf_1E_{nj}^{(2)}(s), \tag{C.0.13}$$
$$C_{nj}^{(1)}(\alpha) = f_2E_{nj}^{(1)}(\alpha)\tau_{nj}^{(1)}(\alpha) - f_1i\alpha, \quad C_{nj}^{(2)}(s) = f_1\tau_{nj}^{(2)}(s) - isf_2E_{nj}^{(2)}(s), \tag{C.0.14}$$
$$D_{nj}^{(1)}(\alpha) = \tau_{nj}^{(1)}(\alpha) - i\alpha E_{nj}^{(1)}(\alpha), \qquad D_{nj}^{(2)}(s) = E_{nj}^{(2)}(s)\tau_{nj}^{(2)}(s) - is, \tag{C.0.15}$$
$$H_{nj}^{(1)}(\alpha) = \sin^2(\theta)B_{nj}^{(1)}(\alpha) + \cos^2(\theta)C_{nj}^{(1)}(\alpha) - \sin(2\theta)D_{nj}^{(1)}(\alpha), \tag{C.0.16}$$
$$J_{nj}^{(1)}(\alpha) = -\sin(\theta)\cos(\theta)B_{nj}^{(1)}(\alpha) + \sin(\theta)\cos(\theta)C_{nj}^{(1)}(\alpha) + \cos(2\theta)D_{nj}^{(1)}(\alpha), \tag{C.0.17}$$

$$F_{nj}^{(2)}(s) = \cos^2(\theta) B_{nj}^{(2)}(s) + \sin^2(\theta) C_{nj}^{(2)}(s) - \sin(2\theta) D_{nj}^{(2)}(s), \tag{C.0.18}$$

$$J_{nj}^{(2)}(s) = \sin(\theta)\cos(\theta)(B_{nj}^{(2)}(s) - C_{nj}^{(2)}(s)) + \cos(2\theta) D_{nj}^{(2)}(s), \tag{C.0.19}$$

$$I_{nj}(s,\alpha,x) = \int_{-\infty}^{\infty} e^{\tau_{nj}^{(2)}(s) y_0 - isx_0 + i\alpha y}, \tag{C.0.20}$$

$$X_{knj}^{(1)}(\alpha) = \begin{cases} H_{nj}^{(1)}(\alpha), & k=1, \\ J_{nj}^{(1)}(\alpha), & k=2, \end{cases} \qquad X_{knj}^{(2)}(s) = \begin{cases} C_{nj}^{(2)}(s), & k=1, \\ D_{nj}^{(2)}(s), & k=2, \\ E_{nj}^{(2)}(s), & k=3, \\ 1, & k=4, \end{cases} \tag{C.0.21}$$

$$\Gamma_{knj}^{(1)}(\alpha, x_n) = \begin{cases} B_{nj}^{(1)}(\alpha) e^{\tau_{nj}^{(1)}(\alpha)x}, & k=1, \\ D_{nj}^{(1)}(\alpha) e^{\tau_{nj}^{(1)}(\alpha)x}, & k=2, \\ E_{nj}^{(1)}(\alpha) e^{\tau_{nj}^{(1)}(\alpha)x}, & k=3, \\ e^{\tau_{nj}^{(1)}(\alpha)x}, & k=4, \end{cases} \qquad L_{knj}^{(2)}(s) = \begin{cases} F_{nj}^{(2)}(s), & k=1, \\ J_{nj}^{(2)}(s), & k=2, \\ E_{nj}^{(2)}(s)\cos(\theta) - \sin(\theta), & k=3, \\ E_{nj}^{(2)}(s)\sin(\theta) + \cos(\theta), & k=4, \end{cases} \tag{C.0.22}$$

where $n = 1,2; j = 1,2,3,4$.

$$B_{tnj}^{(1)}(\alpha) = \left[\frac{f_1 \eta_{nj}^{(1)}(\alpha) \lambda_{nj}^{(1)}(\alpha) - f_2 i\alpha \zeta_{nj}^{(1)}(\alpha)}{d_{nj}^{(1)}(\alpha)} - \frac{1}{k-1} \right] G_{nj}^{(1)}(\alpha), \tag{C.0.23}$$

$$C_{tnj}^{(1)}(\alpha) = \left[\frac{f_2 \eta_{nj}^{(1)}(\alpha) \lambda_{nj}^{(1)}(\alpha) - f_1 i\alpha \zeta_{nj}^{(1)}(\alpha)}{d_{nj}^{(1)}(\alpha)} - \frac{1}{k-1} \right] G_{nj}^{(1)}(\alpha), \tag{C.0.24}$$

$$D_{tnj}^{(1)}(\alpha) = \left[\frac{\zeta_{nj}^{(1)}(\alpha) \lambda_{nj}^{(1)}(\alpha) - i\alpha \eta_{nj}^{(1)}(\alpha)}{d_{nj}^{(1)}(\alpha)} \right] G_{nj}^{(1)}(\alpha), \tag{C.0.25}$$

$$B_{tnj}^{(2)}(s) = \left[\frac{f_2 \zeta_{nj}^{(2)}(s) \lambda_{nj}^{(2)}(s) - is f_1 \eta_{nj}^{(2)}(s)}{d_{nj}^{(2)}(s)} - \frac{1}{k-1} \right] G_{nj}^{(2)}(s), \tag{C.0.26}$$

$$C_{tnj}^{(2)}(s) = \left[\frac{f_1 \zeta_{nj}^{(2)}(s) \lambda_{nj}^{(2)}(s) - is f_2 \eta_{nj}^{(2)}(s)}{d_{nj}^{(2)}(s)} - \frac{1}{k-1} \right] G_{nj}^{(2)}(s), \tag{C.0.27}$$

$$D_{tnj}^{(2)}(s) = \left[\frac{\eta_{nj}^{(2)}(s) \lambda_{nj}^{(2)}(s) - is \zeta_{nj}^{(2)}(s)}{d_{nj}^{(2)}(s)} \right] G_{nj}^{(2)}(s), \tag{C.0.28}$$

$$H^{(1)}_{tnj}(\alpha) = \sin^2(\theta)B^{(1)}_{tnj}(\alpha) + \cos^2(\theta)C^{(1)}_{tnj}(\alpha) - \sin(2\theta)D^{(1)}_{tnj}(\alpha), \tag{C.0.29}$$

$$J^{(1)}_{tnj}(\alpha) = -\sin(\theta)\cos(\theta)(B^{(1)}_{tnj}(\alpha) + C^{(1)}_{tnj}(\alpha)) + \cos(2\theta)D^{(1)}_{tnj}(\alpha), \tag{C.0.30}$$

$$FF^{(2)}_{tnj}(s) = \cos^2(\theta)B^{(2)}_{tnj}(s) + \sin^2(\theta)C^{(2)}_{tnj}(s) - \sin(2\theta)D^{(2)}_{tnj}(s), \tag{C.0.31}$$

$$JJ^{(2)}_{tnj}(s) = \sin(\theta)\cos(\theta)(B^{(2)}_{tnj}(s) - C^{(2)}_{tnj}(s)) + \cos(2\theta)D^{(2)}_{tnj}(s), \tag{C.0.32}$$

$$X^{(1)}_{tknj}(\alpha) = \begin{cases} H^{(1)}_{tnj}(\alpha), & k=1, \\ J^{(1)}_{tnj}(\alpha), & k=2, \end{cases} \qquad X^{(2)}_{tknj}(s) = \begin{cases} C^{(2)}_{tnj}(s), & k=1, \\ D^{(2)}_{tnj}(s), & k=2, \\ \eta^{(2)}_{nj}(s)G^{(2)}_{nj}(s)/d^{(2)}_{nj}(s), & k=3, \\ \zeta^{(2)}_{nj}(s)G^{(2)}_{nj}(s)/d^{(2)}_{nj}(s), & k=4, \end{cases}$$

$$\Gamma^{(1)}_{tknj}(\alpha, x_n) = \begin{cases} B^{(1)}_{tnj}(\alpha)e^{\lambda^{(1)}_{nj}(\alpha)x}, & k=1, \\ D^{(1)}_{tnj}(\alpha)e^{\lambda^{(1)}_{nj}(\alpha)x}, & k=2, \\ \frac{\eta^{(1)}_{tnj}(\alpha)G^{(1)}_{nj}(\alpha)}{d^{(1)}_{nj}(\alpha)}e^{\lambda^{(1)}_{nj}(\alpha)x}, & k=3, \\ \frac{\zeta^{(1)}_{tnj}(\alpha)G^{(1)}_{nj}(\alpha)}{d^{(1)}_{nj}(\alpha)}e^{\lambda^{(1)}_{nj}(\alpha)x}, & k=4, \end{cases} \quad L^{(2)}_{tknj}(s) = \begin{cases} FF^{(2)}_{tnj}(s), & k=1, \\ JJ^{(2)}_{tnj}(s), & k=2 \\ \frac{\eta^{(2)}_{nj}(s)\cos(\theta)-\sin(\theta)\zeta^{(2)}_{nj}(s)}{d^{(2)}_{nj}(s)}G^{(2)}_{nj}(s), & k=3, \\ \frac{\eta^{(2)}_{nj}(s)\sin(\theta)+\cos(\theta)\zeta^{(2)}_{nj}(s)}{d^{(2)}_{nj}(s)}G^{(2)}_{nj}(s), & k=4, \end{cases}$$

where $n = 1, 2; j = 1, 2.$

$$\begin{aligned} M_{nkm}(s, x_0) = \lim_{y_0\to 0^+} \Bigg[& \sum_{j=1}^{2} X^{(2)}_{knj}(s)A^{(2)}_{m+1nj}(s)e^{\tau^{(2)}_{nj}(s)y_0 - isx_0} \\ & + \frac{1}{2\pi}\int_{-\infty}^{\infty} \sum_{j=1}^{4} X^{(1)}_{knj}(\alpha)A_{m+1nj}(\alpha, s)e^{\tau^{(1)}_{nj}(\alpha)x - i\alpha y} d\alpha \Bigg], \end{aligned} \tag{C.0.33}$$

$$R_{nk}(x_0) = -\int_{-\infty}^{\infty} \lim_{y_0\to 0^+} \sum_{j=1}^{4} X^{(1)}_{knj}(\alpha)A_{1nj}(\alpha)e^{\tau^{(1)}_{nj}(\alpha)x - i\alpha y} d\alpha, \tag{C.0.34}$$

$$\begin{aligned} R_{tnk}(x_0) = & -\int_{-\infty}^{\infty} \left[M_{nk1}(s, x_0) + \lim_{y_0\to 0^+} X^{(2)}_{tkn1}(s)W^{(2)}_{n1}(s)e^{\lambda^{(2)}_{n1}(s)y_0 - isx_0} \right]\Phi(s)ds \\ & - \int_{-\infty}^{\infty} \lim_{y_0\to 0^+} \sum_{j=1}^{2} X^{(1)}_{tknj}(\alpha)G^{(1)}_{nj}(\alpha)e^{\lambda^{(1)}_{nj}(\alpha)x - i\alpha y} d\alpha, \end{aligned} \tag{C.0.35}$$

where $n = 1, 2; k = 1, 2; m = 1, 2.$

Appendix D. Some expressions given in Chapter 4.2

$$W_{n1}^{(2)}(s) = \frac{\lambda_{n2}^{(2)}(s)}{\lambda_{n2}^{(2)}(s) - \lambda_{n1}^{(2)}(s)}, \qquad W_{n2}^{(2)}(s) = \frac{\lambda_{n1}^{(2)}(s)}{\lambda_{n2}^{(2)}(s) - \lambda_{n1}^{(2)}(s)}, \tag{D.0.1}$$

$$I_{nj}(s, \alpha, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(\lambda_{nj}^{(2)}(s) y_1 - isx_1 + i\alpha y) dy, \tag{D.0.2}$$

$$FF_{nj}^{(2)}(s) = -\lambda_{nj}^{(2)}(s) \sin(\theta) - is\cos(\theta), \tag{D.0.3}$$

$$\begin{aligned} K_n(x_1, s) = &\lim_{y_1 \to 0^+} \int_{-\infty}^{\infty} \sum_{j=1}^{2} AA_j^{'}(\alpha, s) FF_{nj}^{(1)}(\alpha) \ \exp(\lambda_{nj}^{(1)}(\alpha) x - i\alpha y) d\alpha \\ &+ \left[W_{n1}^{(2)}(s) \lambda_{n1}^{(2)}(s) - \frac{2\pi h_c}{k_n(x_1, 0^+)} \right] e^{-isx_1}, \end{aligned} \tag{D.0.4}$$

$$R_n(x_1) = -\lim_{y_1 \to 0^+} \int_{-\infty}^{\infty} \sum_{j=1}^{2} A_j^{'}(\alpha) FF_{nj}^{(1)}(\alpha) \ \exp(\lambda_{nj}^{(1)}(\alpha) x - i\alpha y) d\alpha, \tag{D.0.5}$$

$$A_j^{'}(\alpha) = \begin{cases} A_j(\alpha), & n = 1, \\ A_{j+2}(\alpha), & n = 2, \end{cases} \qquad AA_j^{'}(\alpha, s) = \begin{cases} AA_j(\alpha, s), & n = 1, \\ AA_{j+2}(\alpha, s), & n = 2, \end{cases} \tag{D.0.6}$$

$$FF_{nj}^{(1)}(\alpha) = -\lambda_{nj}^{(1)}(\alpha) \sin(\theta) - i\alpha\cos(\theta), \qquad n = 1, 2, \tag{D.0.7}$$

$$E_{nk}(x_1) = B_k G_k \int_{-\infty}^{\infty} \left(\int_{-a_0}^{x_1} K_n(x_1, s) \ dx_1 \right) \frac{J_{k+1}(sa_0)}{s} \ ds, \qquad S_n(x_1) = \int_{-a_0}^{x_1} R_n(x_1) \ dx_1 \tag{D.0.8}$$

where $n = 1, 2; j = 1, 2$.

$$C^{(1)}_{nj}(\alpha) = \frac{(k-1)\tau^{(1)}_{nj}(\alpha)(\tau^{(1)}_{nj}(\alpha)+\beta_n) - (1+k)\alpha^2}{i\alpha(2\tau^{(1)}_{nj}(\alpha)+(k-1)\beta_n)}, \quad n = 1, 2; j = 1, 2, 3, 4, \tag{D.0.9}$$

$$d^{(1)}_{nj}(\alpha) = (1-k^2)\left[\lambda^{(1)}_{nj}(\alpha)(\lambda^{(1)}_{nj}(\alpha)+\beta_n)(\lambda^{(1)}_{nj}(\alpha)^2+\lambda^{(1)}_{nj}(\alpha)\beta_n-2\alpha^2)+\alpha^2\left(\alpha^2+f_0^2\beta_n^2\right)\right], \tag{D.0.10}$$

$$\begin{aligned} u^{(1)}_{tnj}(\alpha) = \Bigg[&-\left(\gamma_n+\beta_n+\lambda^{(1)}_{nj}(\alpha)\right)\left((k-1)\lambda^{(1)}_{nj}(\alpha)(\lambda^{(1)}_{nj}(\alpha)+\beta_n)-(1+k)\alpha^2\right) \\ &-\alpha^2\left(2\lambda^{(1)}_{nj}(\alpha)+\beta_n(3-k)\right)\Bigg]\frac{A^{(1)}_{nj}(\alpha)}{d^{(1)}_{nj}(\alpha)}, \end{aligned} \tag{D.0.11}$$

$$\begin{aligned} v^{(1)}_{tnj}(\alpha) = \Bigg[&\left((1+k)\lambda^{(1)}_{nj}(\alpha)(\lambda^{(1)}_{nj}(\alpha)+\beta_n)-(k-1)\alpha^2\right) \\ &-\left(2\lambda^{(1)}_{nj}(\alpha)+\beta_n(k-1)\right)\left(\gamma_n+\beta_n+\lambda^{(1)}_{nj}(\alpha)\right)\Bigg]\frac{i\alpha A^{(1)}_{nj}(\alpha)}{d^{(1)}_{nj}(\alpha)}, \end{aligned} \tag{D.0.12}$$

where $n = 1, 2; j = 1, 2$.

$$C^{(2)}_{nj}(s) = -\left[\frac{(1+k)\tau^{(2)}_{nj}(s)(\tau^{(2)}_{nj}(s)+\beta_{n2})-(k-1)s(s+i\beta_{n1})}{(k-1)\beta_{n1}\tau^{(2)}_{nj}(s)-is(2\tau^{(2)}_{nj}(s)+(3-k)\beta_{n2})}\right], \ n = 1, 2; j = 1, 2, 3, 4, \tag{D.0.13}$$

$$d^{(2)}_{nj}(s) = (1-k^2)\Big[f_0^2(s\beta_{n2}-i\beta_{n1}\lambda^{(2)}_{nj}(s))^2+(\lambda^{(2)}_{nj}(s)(\lambda^{(2)}_{nj}(s)+\beta_{n2})-s(s+i\beta_{n1}))^2\Big], \tag{D.0.14}$$

$$\begin{aligned} u^{(2)}_{tnj}(s) = \Bigg[&\left(\gamma_{n2}+\beta_{n2}+\lambda^{(2)}_{nj}(s)\right)\left(((3-k)\beta_{n1}-2is)\lambda^{(2)}_{nj}(s)-is(k-1)\beta_{n2}\right) \\ &-(\gamma_{n1}+\beta_{n1}-is)\left((1+k)\lambda^{(2)}_{nj}(s)(\lambda^{(2)}_{nj}(s)+\beta_{n2})-(k-1)s(s+i\beta_{n1})\right)\Bigg]\frac{A^{(2)}_{nj}(s)}{d^{(2)}_{nj}(s)}, \end{aligned} \tag{D.0.15}$$

$$v^{(2)}_{tnj}(s) = \Bigg[(\gamma_{n1}+\beta_{n1}-is)\left((k-1)\beta_{n1}\lambda^{(2)}_{nj}(s)-is(3-k)\beta_{n2}-2is\lambda^{(2)}_{nj}(s)\right)$$

$$- \left(\lambda_{nj}^{(2)}(s) + \gamma_{n2} + \beta_{n2}\right)\left((k-1)\lambda_{nj}^{(2)}(s)(\lambda_{nj}^{(2)}(s) + \beta_{n2}) - s(1+k)(s + i\beta_{n1})\right)\Bigg] \frac{A_{nj}^{(2)}(s)}{d_{nj}^{(2)}(s)}, \tag{D.0.16}$$

$$D_{nj}^{(2)}(s) = \left[(1+k)\tau_{nj}^{(2)}(s) - is(3-k)C_{nj}^{(2)}(s)\right]/(k-1), \tag{D.0.17}$$

$$F_{nj}^{(2)}(s) = \left[(3-k)\tau_{nj}^{(2)}(s) - is(1+k)C_{nj}^{(2)}(s)\right]/(k-1), \tag{D.0.18}$$

$$E_{nj}^{(2)}(s) = \left[C_{nj}^{(2)}(s)\tau_{nj}^{(2)}(s) - is\right], \tag{D.0.19}$$

$$G_{nj}^{(2)}(s) = \sin^2(\theta)D_{nj}^{(2)}(s) - \sin(2\theta)E_{nj}^{(2)}(s) + \cos^2(\theta)F_{nj}^{(2)}(s), \tag{D.0.20}$$

$$H_{nj}^{(2)}(s) = \sin(\theta)\cos(\theta)(-D_{nj}^{(2)}(s) + F_{nj}^{(2)}(s)) + \cos(2\theta)E_{nj}^{(2)}(s), \tag{D.0.21}$$

$$J_{nj}^{(2)}(s) = \cos(\theta)C_{nj}^{(2)}(s) - \sin(\theta), \qquad K_{nj}^{(2)}(s) = \sin(\theta)C_{nj}^{(2)}(s) + \cos(\theta) \tag{D.0.22}$$

$$D_{nj}^{(1)}(\alpha) = \left[(3-k)C_{nj}^{(1)}(\alpha)\tau_{nj}^{(1)}(\alpha) - i\alpha(1+k)\right]/(k-1), \tag{D.0.23}$$

$$F_{nj}^{(1)}(\alpha) = \left[(1+k)C_{nj}^{(1)}(\alpha)\tau_{nj}^{(1)}(\alpha) - i\alpha(3-k)\right]/(k-1), \tag{D.0.24}$$

$$E_{nj}^{(1)}(\alpha) = \left[\tau_{nj}^{(1)}(\alpha) - i\alpha C_{nj}^{(1)}(\alpha)\right], \tag{D.0.25}$$

$$G_{nj}^{(1)}(\alpha) = \cos^2(\theta)D_{nj}^{(1)}(\alpha) - \sin(2\theta)E_{nj}^{(1)}(\alpha) + \sin^2(\theta)F_{nj}^{(1)}(\alpha), \tag{D.0.26}$$

$$H_{nj}^{(1)}(\alpha) = \sin(\theta)\cos(\theta)(D_{nj}^{(1)}(\alpha) - F_{nj}^{(1)}(\alpha) + \cos(2\theta)E_{nj}^{(1)}(\alpha), \tag{D.0.27}$$

$$S_{nj}(s,\alpha,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(\tau_{nj}^{(2)}(s)y_1 - isx_1 + i\alpha y)dy, \tag{D.0.28}$$

where $n = 1, 2; j = 1, 2, 3, 4$.

$$D_{nj}^{'(2)}(s) = \left[(1+k)v_{tnj}^{(2)}(s)\lambda_{nj}^{(2)}(s) - is(3-k)u_{tnj}^{(2)}(s) - A_{nj}^{(2)}(s)\right]/(k-1), \tag{D.0.29}$$

$$F_{nj}^{'(2)}(s) = \left[(3-k)v_{tnj}^{(2)}(s)\lambda_{nj}^{(2)}(s) - is(1+k)u_{tnj}^{(2)}(s) - A_{nj}^{(2)}(s)\right]/(k-1), \tag{D.0.30}$$

$$E_{nj}^{'(2)}(s) = \left[u_{tnj}^{(2)}(s)\lambda_{nj}^{(2)}(s) - isv_{tnj}^{(2)}(s)\right], \tag{D.0.31}$$

$$G_{nj}^{'(2)}(s) = \sin^2(\theta)D_{nj}^{'(2)}(s) - \sin(2\theta)E_{nj}^{'(2)}(s) + \cos^2(\theta)F_{nj}^{'(2)}(s), \tag{D.0.32}$$

$$H_{nj}^{'(2)}(s) = \sin(\theta)\cos(\theta)(-D_{nj}^{'(2)}(s) + F_{nj}^{'(2)}(s)) + \cos(2\theta)E_{nj}^{'(2)}(s), \tag{D.0.33}$$

$$J_{nj}^{'(2)}(s) = \cos(\theta)u_{tnj}^{(2)}(s) - \sin(\theta)v_{tnj}^{(2)}(s), \qquad K_{nj}^{'(2)}(s) = \sin(\theta)u_{tnj}^{(2)}(s) + \cos(\theta) \tag{D.0.34}$$

$$D_{nj}^{'(1)}(\alpha) = \left[(3-k)u_{tnj}^{(1)}(\alpha)\lambda_{nj}^{(1)}(\alpha) - i\alpha(1+k)v_{tnj}^{(1)}(\alpha) - A_{nj}^{(1)}(\alpha)\right]/(k-1), \tag{D.0.35}$$

$$F_{nj}^{'(1)}(\alpha) = \left[(1+k)u_{tnj}^{(1)}(\alpha)\lambda_{nj}^{(1)}(\alpha) - i\alpha(3-k)v_{tnj}^{(1)}(\alpha) - A_{nj}^{(1)}(\alpha)\right]/(k-1), \tag{D.0.36}$$

$$E_{nj}^{'(1)}(\alpha) = \left[v_{tnj}^{(1)}(\alpha)\lambda_{nj}^{(1)}(\alpha) - i\alpha u_{tnj}^{(1)}(\alpha)\right], \tag{D.0.37}$$

$$G_{nj}^{'(1)}(\alpha) = \cos^2(\theta)D_{nj}^{'(1)}(\alpha) - \sin(2\theta)E_{nj}^{'(1)}(\alpha) + \sin^2(\theta)F_{nj}^{'(1)}(\alpha), \tag{D.0.38}$$

$$H_{nj}^{'(1)}(\alpha) = \sin(\theta)\cos(\theta)(D_{nj}^{'(1)}(\alpha) - F_{nj}^{'(1)}(\alpha) + \cos(2\theta)E_{nj}^{'(1)}(\alpha), \tag{D.0.39}$$

where $n = 1, 2; j = 1, 2$.

$$FE_{mnj}^{(1)}(\alpha) = \begin{cases} F_{nj}^{(1)}(\alpha), & m = 1, \\ E_{nj}^{(1)}(\alpha), & m = 2, \end{cases} \qquad C1_{mnj}^{(1)}(\alpha) = \begin{cases} C_{nj}^{(1)}(\alpha), & m = 1, \\ 1, & m = 2, \end{cases}$$

Similarly for $FE_{mnj}^{'(1)}(\alpha)$, $GH_{mnj}^{(2)}(s)$, $GH_{mnj}^{'(2)}(s)$, $JK_{mnj}^{(2)}(s)$, $JK_{mnj}^{'(2)}(s)$, $uv_{tmnj}^{(1)}(\alpha)$, $DE_{mnj}^{(2)}(s)$, $DE_{mnj}^{'(2)}(s)$, $GH_{mnj}^{(1)}(\alpha)$ and $GH_{mnj}^{'(1)}(\alpha)$ $(m = 1, 2)$.

$$\begin{aligned} N_{nmk}(x_1, s) = &\sum_{j=1}^{4} B_{k+1\ nj}^{(2)}(s)DE_{mnj}^{(2)}(s)\exp(-isx_1) \\ &+ \lim_{y_1 \to 0^+} \int_{-\infty}^{\infty} BB_j^{'}(\alpha, s)GH_{mnj}^{(1)}(\alpha)\exp(\tau_{nj}^{(1)}(\alpha)x - i\alpha y)d\alpha, \end{aligned} \tag{D.0.40}$$

$$\begin{aligned} P_{mn}(x_1) = &\lim_{y_1 \to 0^+} \int_{-\infty}^{\infty} \left[\sum_{j=1}^{4} B_j^{'}(\alpha)GH_{mnj}^{(1)}(\alpha)\exp(\tau_{nj}^{(1)}(\alpha)x) - 4\alpha_n(x_1, 0)\sum_{j=1}^{2} GH_{mnj}^{'(1)}(\alpha)\right. \\ &\times \left.\exp(\lambda_{nj}^{(1)}(\alpha)x)\right]\exp(-i\alpha y)d\alpha - \int_{-\infty}^{\infty}\left[\sum_{j=1}^{4} B_{1nj}^{(2)}(s)DE_{mnj}^{(2)}(s) + 4\alpha_n(x_1, 0)\right. \\ &\times \left.\sum_{j=1}^{2} DE_{mnj}^{'(2)}(s)\right]\exp(-isx_1)ds, \end{aligned} \tag{D.0.41}$$

$$B_j^{'}(\alpha) = \begin{cases} B_j(\alpha), & n = 1, \\ B_{j+4}(\alpha), & n = 2, \end{cases} \qquad BB_j^{'}(\alpha, s) = \begin{cases} BB_j(\alpha, s), & n = 1, k = 1, \\ BB_{j+4}(\alpha, s), & n = 2, k = 1, \\ BBB_j(\alpha, s), & n = 1, k = 2, \\ BBB_{j+4}(\alpha, s), & n = 2, k = 2. \end{cases} \tag{D.0.42}$$

where $n, m, k = 1, 2$.

List of Publications

1. **Singh, R., & Das, S.** (2021). Investigation of interactions among collinear Griffith cracks situated in a functionally graded medium under thermo-mechanical loading. ***Journal of Thermal Stresses***, 44, 433- 455. *I.F : 3.456*

2. **Singh, R., & Das, S.** (2021). Transient response of collinear Griffith cracks in a functionally graded strip bonded between dissimilar elastic strips under shear impact loading. ***Composite Structures***, 263, 113635. *I.F : 6.603*

3. **Singh, R., & Das, S.** (2021). Mathematical study of an arbitrary-oriented crack crossing the interface of bonded functionally graded strips under thermo-mechanical loading. ***Theoretical and Applied Fracture Mechanics***, 103170. *I.F: 4.374*

4. **Singh, R., & Das, S.** (2022). Schmidt method to study the disturbance of steady-state heat flows by an arbitrary oriented crack in bonded functionally graded strips. ***Composite Structures***, 287, 115329. *I.F : 6.603*

5. **Tanwar, A., Singh, R., & Das, S.** (2022). Interaction among offset parallel cracks in an orthtropic plane under thermo-mechanical loading. ***Journal of Applied Mathematics and Mechanics***, 102, e202100593. *I.F : 1.759*

6. **Singh, R., & Das, S.** (2023). Analysis of multiple parallel cracks in a functionally graded magneto-electro-elastic plane using boundary collocation method. ***Archive of Applied Mechanics***, 93, 4497–4516. *I.F : 2.467*

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